Quality is in the Eye of the Beholder: Taste Projection in Markets with Observational Learning*

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June 16, 2023

Abstract

We study how misperceptions of others' tastes influence beliefs, demand, and prices in markets with observational learning. Consumers infer the commonly-valued quality of a good from the quantity demanded and price paid by others. When consumers exaggerate the similarity between their and others' tastes, such "taste projection" leads to discrepant quality perceptions across consumers. A projector's (mis)inferred quality is negatively related to their taste and increasing in the observed price. These biased beliefs generate an excessively elastic market demand. We also analyze dynamic monopoly pricing with short-lived taste-projecting consumers. Optimal pricing follows a declining path: the seller uses a high price early to inflate future buyers' perceptions, and then lowers it gradually to induce over-adoption.

JEL Classification: D42; D82; D83; D91.
Keywords: Social Learning; Dynamic Pricing; Projection Bias; False-Consensus Effect.

*For helpful comments, we thank Yair Antler, Benjamin Bushong, Rahul Deb, Erik Eyster, Simone Galperti, Alessandro Lizzeri, Nima Haghpanah, Matthew Rabin, Larry Samuelson, Joshua Schwartzstein, Sevgi Yuksel and the audiences at the Australasian Local Economic Theory Seminar (ALETS), the 2021 Australasian Economic Theory Workshop at the University of Sydney, UTS, UCSB, Purdue, University of Queensland, Florida State, Boston College, Università degli Studi di Napoli Federico II, University of Manchester, the 2021 Melbourne IO Workshop, the 2021 Asia Pacific Industrial Organization Conference in Singapore, the 2022 IIOC in Boston, the 2022 CSEF-IGIER Symposium on Economics and Institutions in Anacapri, the Workshop on Decision Making under Uncertainty at St Leo's College (UQ), CEPR Virtual IO Seminar Series, the 2022 Stony Brook International Conference on Game Theory, SAET 2022 in Canberra, and the 2022 Latin American Workshop in Economic Theory.
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1 Introduction

We often use the popularity of a product to assess its quality. We may naturally expect, for instance, that a new electric car has better performance when more people buy it, that a new health trend provides greater benefits when more of our friends adopt it, or that an investment has a higher expected return when our colleagues flock to it. Indeed, a large theoretical and empirical literature has emphasized how observational learning shapes the adoption of new products, spanning consumer goods, entertainment, insurance plans, agricultural technologies, and financial products (see e.g., Mobius and Rosenblat, 2014 and Bikhchandani et al., 2021 for reviews).

But how does social learning operate when people don't fully appreciate how others' preferences differ from their own? In all the examples above, choices are not driven purely by perceptions of a commonly-valued quality, but also depend on idiosyncratic tastes and motives. For instance, some consumers driving electric vehicles might have a distinct desire to reduce their carbon footprints; and some people investing in cryptocurrencies might be more risk tolerant than others. Yet, do consumers and investors properly account for the fact that others' choices reflect private information as well as their tastes? Long-standing literatures in psychology on social projection and the false-consensus effect, along with mounting evidence from economics, suggest the answer is no. In particular, people often exaggerate the degree to which others' tastes are similar to their own (Ross et al., 1977; Marks and Miller, 1987; Krueger and Clement, 1994; Engelmann and Strobel, 2012). For example, those with specific tastes for certain consumer products tend to overestimate how many share these tastes (Orhun and Urminsky, 2013). Such misperceptions also arise when evaluating others' risk preferences (Faro and Rottenstreich, 2006), political preferences (Delavande and Manski, 2012), and taste for effort (Bushong and Gagnon-Bartsch, 2023a). Moreover, a recent meta-analysis by Bursztyn and Yang (2022) shows that misperceptions of others are widespread in the field, underscoring the importance of further understanding their market implications.

In this paper, we study how such "taste projection" distorts consumers' beliefs, market demand, and prices in a social-learning environment where valuations for a product have both a common and private component. The common component—the product's intrinsic quality—is initially unknown to (some) consumers, who try to infer it from the quantity demanded by others at a given price. Consumers know the private component of their valuation (i.e., their idiosyncratic taste for the product), but wrongly "project" this onto others: they exaggerate how similar others' tastes are to their own. We show that taste projection leads consumers to systematically mislearn a product's quality, characterizing how these biased beliefs depend on an individual's own taste and the observed price, and how they ultimately shape market demand. Furthermore, we analyze the optimal pricing strategy of a seller who is aware of consumers' projection.

Our implications are particularly relevant for markets where consumers with heterogeneous tastes actively rely on others' choices to guide their own—e.g., those with prominent best-seller lists or a tendency to trend on social media. For instance, consider the health and wellness industry, where new products—whose quality is ex-ante uncertain and difficult to ascertain—are routinely introduced; e.g., novel workout equipment, “innovative” fitness classes, or “revolutionary” dietary regimens.[1] Consumers’ willingness to pay for such products and services is of course influenced by their perceived health benefits; yet, consumers likely differ in their idiosyncratic tastes for exercise or a particular diet.

For a concrete example, consider Inês and Peter who are independently contemplating whether to enroll in a fitness program touting some of these new features. Inês enjoys an active lifestyle; Peter does not, but his physician has encouraged him to get in shape. Suppose they both see an article reporting the number of people who joined the program in the past six months. Projection will lead them to draw different inferences about the program’s potential benefits. Projecting her love of fitness onto others, Inês will find the take-up rate disappointingly low. Conversely, the number of adopters will look very high to Peter. Hence, they draw conflicting conclusions despite observing exactly the same information—inferred quality is “in the eye of the beholder.” In particular, Inês, who *likes* exercise, forms a more pessimistic inference. Taste projection therefore induces consumers with a stronger idiosyncratic taste for a product to inadvertently be more critical when judging its quality from its popularity. By contrast, Peter becomes too eager to join the program, exaggerating its benefits and potentially over-consuming in various ways (e.g., enrolling in unnecessary classes or subscribing to an unproven diet plan).

Moreover, because Inês and Peter’s inferences are negatively related to their idiosyncratic taste, their (perceived) total valuations for the program will be too similar. Although the difference between these valuations should be driven solely by the difference in their private values, Inês’s pessimistic inference deflates her total perceived valuation, whereas the opposite holds for Peter. Hence, taste projection is self-fulfilling: because buyers believe that idiosyncratic tastes are less dispersed than they actually are, they will draw divergent inferences about the common value in a way that results in total (perceived) valuations that are indeed less dispersed.

While the direction of Inês’s and Peter’s misinference will depend on their specific tastes, a more subtle implication of taste projection is that they will both form inferences that increase in the program’s price, irrespective of their taste. Indeed, because projectors underestimate the heterogeneity in others’ valuations, they believe market demand is more elastic than it really is. Therefore, although they correctly predict the take-up rate to decrease with the program’s price, they expect to see even fewer patrons than what a rational consumer would predict as the price increases. To rationalize this higher-than-expected demand, they will conclude that quality is higher when the price is higher. More broadly, projectors systematically overestimate the quality of a product when they see others buying it at a price they themselves are not initially willing to pay since they over-attribute

[1] For instance, the New York Times has noted a growing demand for high-end fitness clubs (“Think Getting Into College Is Hard? Try Applying for These Gyms”: www.nytimes.com/2023/03/25/style/exclusive-gym-memberships.html).

others' purchases to positive information rather than differences in tastes. Hence, projection provides a simple yet novel explanation for why quality perceptions are often swayed by prices.[2]

The properties of misinference described above create new incentives for a seller that would not arise under rational learning. First, since perceived quality increases in the observed price, there is a "belief-manipulation effect": in a dynamic setting, a monopolist will set high prices early on to inflate future consumers' beliefs about the value of its product. This holds even when consumers think the seller does not have an informational advantage, and hence it is not driven by classical signaling motives. Second, projectors' perceived valuations being excessively similar creates an "elasticity effect": projectors' demand is more elastic than that of rational agents, and thus a reduction in the current price has a greater effect in attracting new consumers. Together, these effects imply that a monopolist's optimal pricing strategy follows a declining path. The seller uses high prices in earlier periods to inflate later consumers' quality perceptions (i.e., creating "hype"), and then reaps the benefits of such manipulation by gradually lowering the price to induce over-adoption.[3]

We present our model in Section 2. In each period n, a new generation of consumers enters the market and decides whether to buy a product with an uncertain quality, $\omega \in \mathbb{R}$. Each consumer i's valuation for the product is increasing in both ω and their private value, or "taste," t_i. Some consumers observe a signal s correlated with ω while others are uninformed and rely on social learning to estimate ω. In particular, we assume that individuals observe the quantity demanded and price from the previous round. We focus on a setting with a continuum of consumers acting in each period, which allows rational observers to perfectly infer their predecessors' signal. This provides a simple environment to study the effects of taste projection, since any learning failures arise from projection itself rather than other frictions to information aggregation.

Our model of taste projection adapts Gagnon-Bartsch et al.'s (2021a) more general model to our setting. Individuals hold misspecified models about the distribution of tastes: private values are in fact independently drawn from a distribution F, yet an individual with private value t_i mistakes F for a distribution $\widehat{F}(\cdot|t_i)$ that is overly concentrated around his own value, t_i. We close the model with a solution concept in which individuals are naive about their own bias and that of others, but are otherwise rational. Hence, each individual i believes he faces an environment in which all players agree that private values are distributed according to $\widehat{F}(\cdot|t_i)$.

We begin our analysis in Section 3 by studying a static version of our model with a fixed price.

[2]A large literature, primarily in marketing, has documented a positive relationship between prices and perceived quality; see Monroe (1973) and Rao and Monroe (1989) for early reviews and Völckner and Hofmann (2007) for a more recent one. This perceived relationship emerges even in settings where the true relationship between price and quality is weak or non-existent (e.g., Gerstner, 1985 and Broniarczyk and Alba, 1994) and is strengthened in settings where, as in our model, people observe others' purchase decisions (Yan and Sengupta, 2011).

[3]In this way, we provide a novel explanation for why advertising high previous prices can persuade consumers to buy at a new lower price. This contrasts with other explanations based on salience (e.g,. Bordalo et al., 2013, 2020) or an intrinsic "taste for bargains" (e.g., Jahedi, 2011; Armstrong and Chen, 2020), and it arises even when prices do not rationally signal quality (as in, e.g., Bagwell and Riordan, 1991 or Taylor, 1999).

The purpose is twofold. First, it allows us to simply highlight comparative statics that are fundamental for understanding how biased beliefs evolve in the dynamic case. Furthermore, since the static model can be seen as the steady-state of our dynamic one, this analysis establishes that these comparative statics are not merely short-run effects, but are also robust steady-state phenomena. The steady-state analysis reflects the logic of a rational-expectations equilibrium (Grossman, 1976; Grossman and Stiglitz, 1980), albeit with agents forming diverse and misspecified expectations.

As foreshadowed by our example, taste projection has three main effects. First, a consumer's perceived quality is negatively related to his taste. That is, when his private value is higher, he expects the good to be more attractive to others, which makes him more pessimistic about its inferred quality.[4] Second, each buyer's perceived quality is increasing in the equilibrium price. Indeed, because he underestimates the heterogeneity in tastes, a projector's conjectured demand curve is a counter-clockwise rotation of the true one and, as a result, he exaggerates the local demand elasticity. If the price were to increase, then the quantity demanded would fall by less than what a projector would predict under the beliefs he formed at the original price; hence, the projectors' beliefs about quality must also increase to compensate for a less-than-predicted drop in quantity demanded. Third, projecting agents' perceived total valuations are less dispersed than under rational learning. Although a buyer with a high private value (i.e., a "high type") perceives a greater benefit from adoption than a low type, the wedge between these perceptions is diminished relative to the rational benchmark. This results in a market demand that is excessively elastic.

How would a profit-maximizing monopolist set prices over time to exploit these biases? We turn to this question in Section 4, where we analyze our dynamic model. We begin with two results building on the intuitions above. First, we show that a projector's inferred quality is increasing in *all* past prices. Second, we show that consumers' demand overreacts to a price change: a price cut attracts too many consumers since it moves the marginal buyer into the region of types who overestimate quality, whereas a price hike excludes too many for the opposite reason.

More generally, projection induces an intertemporal link in the seller's pricing incentives that is absent under rational inference. In our simple environment, the optimal strategy under rational learning is to continually charge the static monopoly price. With projection, however, the seller prefers a decreasing price path. This results from a balancing of the effects described above in the static model, which analogously emerge in the dynamic case. On the one hand, since demand overreacts to price changes, undercutting the previous price would attract a magnified mass of consumers in the current period. On the other hand, increasing the current price boosts the perceived quality of future consumers at the cost of forgoing current sales. The seller's pricing strategy optimally

[4] Bushong and Gagnon-Bartsch (2023b) find evidence of this prediction in a social-learning experiment where subjects learn about the nominal value of gift cards to various businesses by observing the choices of privately-informed others. Those subjects who enjoy a given business more form more pessimistic inferences about the value of the gift card. Relatedly, Tucker and Zhang (2011) study data from a website listing wedding-service vendors and find that, fixing the level of vendor popularity, customers infer lower quality the broader is the appeal of the vendor.

balances these effects by setting an inflated initial price above the static monopoly price and then gradually reducing it. High initial prices inflate future consumers beliefs, while also providing scope to reduce prices over time and capitalize on consumers' overreaction to price cuts.[5]

Our analysis of optimal pricing first considers the two-period case. There, we show that a high initial price followed by a lower price is a general feature of our model. We also show that projection increases the seller's profit and discuss its effect on consumer welfare. Although the expansion of sales in the second period can shrink the traditional deadweight loss associated with monopoly, projection can introduce new forms of inefficiency. Since low types tend to overestimate quality, they are systematically lured into buying even when they should not. Indeed, the seller's manipulative pricing induces excessive take-up among uninformed buyers, consistent with notions of herding or bandwagon effects. Moreover, when projection is sufficiently strong, even consumers with *negative* valuations can be induced to buy the good. We then consider longer horizons, focusing on the particularly tractable case of uniformly distributed tastes and show that a declining price path—with an initial price above the rational monopoly price—emerges for a horizon of any arbitrary length.

Section 5 analyzes two extensions of our model. First, we consider a two-period setting with "long-lived" consumers who can buy in either period, and we show that projectors still over-adopt the good even when the price is fixed. A selection effect naturally emerges: high types buy in the first period whereas uninformed low types delay in order to glean information from initial adopters. Projectors who delay under-appreciate this selection effect, since they underestimate the taste difference between early adopters and themselves. Thus, they overestimate quality when observing high initial demand, which causes too many to buy and generates systematic disappointment among those who do.[6] Second, we revisit the static equilibrium from Section 3 but allow for multi-unit demand. Since perceived quality is negatively related to taste, projectors with a strong taste for the product will under-consume while those with a weak taste will over-consume. Thus, all projectors experience inefficiencies, and those with more extreme tastes suffer more.

Section 6 concludes by discussing some additional applications of our framework. We suspect that taste projection may have important consequences for how people value their information sources. For instance, suppose that individuals entertain the possibility that others are biased in favor of a particular option (e.g., a brand or politician), supporting it regardless of their information. Even when such blind support is absent in reality, projectors are prone to think it exists. For example, a

[5]This manipulating role of high initial prices is reminiscent of other signaling strategies discussed in the literature. For instance, Stock and Balachander (2005) show that a monopolist might choose to make a product scarce in order to signal its quality to uninformed consumers; similarly, Miklós-Thal and Zhang (2013) argue that in the early life of a product, "demarketing" strategies that discourage consumers (e.g., limited advertising, understocking inventory) can raise the product's perceived quality. Compared to this literature, we emphasize a different mechanism through which restraining initial sales via high prices can inflate later consumers' quality perceptions.

[6]Empirical studies show that second-wave consumers tend to display greater dissatisfaction and suggest that this may stem from selection neglect (e.g., Li and Hitt, 2008; Dai et al., 2018). Our model provides a specific mechanism explaining why consumers may under-appreciate these selection effects.

projector who despises an option will see far too many people (in her eyes) supporting it. To explain this discrepancy, she may come to believe that others' have some ulterior motive, neglecting that their support may come from mere differences in taste. Such skepticism of others' motives may lead people to discredit others' actions, which may shed light on why some factions are unmoved by others' actions even when they reveal valuable information.

Related Literature

We contribute to a recent literature exploring how specific behavioral biases, along with more general forms of model misspecification, interfere with social learning. Much of this literature considers environments similar to the sequential "herding" models of Banerjee (1992), Bikhchandani et al. (1992), and Smith and Sørensen (2000) and studies when long-run beliefs may converge on a false state of the world or fail to converge at all. For instance, Eyster and Rabin (2010), Bohren (2016), and Gagnon-Bartsch and Rabin (2021) examine how neglecting the redundancy of information in others' actions can lead society to grow convinced of a false state. Bohren and Hauser (2021) and Frick et al. (2023) provide frameworks for studying the convergence of beliefs under a wide range of misspecified models. Closer to the specific error we study, Frick et al. (2020) show that when agents share a common misperception of the type distribution, even small amounts of misspecification can cause incorrect learning. Gagnon-Bartsch (2016) considers a simple variant of taste projection with two types who hold conflicting misperceptions, showing how it can cause different types to grow confident in distinct states or generate beliefs that perpetually cycle. In contrast, instead of asking whether information aggregates in the long-run, we study the comparative statics of projectors' erroneous beliefs in cases where they necessarily mislearn. Moreover, unlike the papers above, we focus on market outcomes when prices explicitly influence agents' beliefs and examine how a sophisticated seller would optimally use prices to strategically distort those beliefs.

In this way, we also contribute to a literature in IO on pricing in the presence of observational learning, which has largely focused on rational inference. This literature mainly considers settings with frictions to information aggregation, analyzing how a seller's behavior either alleviates or intensifies these frictions. For instance, Bose et al. (2006, 2008) consider a pure common-value environment with a long-lived monopolist who, in each period, faces an uninformed, short-lived buyer. Buyers learn about the common value from the history of prices and their predecessors' purchase decisions. Information aggregates slowly because there is a single buyer in each period with a discrete signal, and the seller maximizes profits by setting prices that reveal as much information as possible.Using a similar setting, Parakhonyak and Vikander (2023) show that a monopolist may want to strategically create product scarcity in order to trigger a "buying herd." More similar to our setup, Caminal and Vives (1996, 1999) consider a model with a continuum of short-lived consumers who are privately but imperfectly informed about the quality of two competing products. Consumers in a later generation don't observe past prices, but try to infer a product's quality from its market share in the previous period. Differently from us, because consumers cannot see the previous price, sellers set low introductory prices to boost sales in an attempt to convince buyers that their quality is high.

The IO literature described above mainly focuses on cases where the seller does not have an informational advantage over buyers, and we follow in this tradition. However, another related strand of this literature examines how a privately informed seller can signal the quality of its good through prices and other means. While we do not analyze such signaling, some of our predictions resemble those from this literature. For instance, Bagwell and Riordan (1991) analyze a monopolistic market with a mix of informed and uninformed consumers (like us), and show that high and declining prices can signal higher quality to uninformed consumers when the high-quality seller has a sufficiently high cost. In contrast, our mechanism generates quality perceptions that are increasing in price even when a seller's quality is not tightly linked to their costs.[7] Taylor (1999) considers a two-period model with private and common values where a seller is privately informed about the quality of its house, and short-lived buyers try to learn it from the time the house is on the market. The optimal price path is declining since a higher first-period price sends a less negative signal when the house is not sold. At a broader level, we differ from both strands above by considering a setting that neutralizes the informational frictions that impede rational learning (e.g., incomplete learning, search costs, or signaling motives) in order to isolate how taste projection itself interferes with learning.

There is also a small but growing theoretical literature on the implications of taste projection and the false-consensus effect in domains different from ours. Goeree and Grosser (2007) examine how a false-consensus effect can lead to inefficient election outcomes, while Frick et al. (2022) show how the false-consensus effect may arise when agents neglect the assortative nature of matching when interacting with others. Gagnon-Bartsch et al. (2021a) show how projection of private values can lead to overbidding and inefficient allocations in auctions. Relatedly, Madarász (2012, 2021) studies the implications of "information projection"—whereby agents exaggerate the extent to which others know their private information—in several contexts (e.g., communication and bargaining).

2 Model

In this section, we introduce the basic features of the environment and present our model of taste projection. Subsequent sections examine projection in various settings (e.g., static versus dynamic); we will describe the specific features of those settings in their respective sections, but we introduce their common core here.

[7]The optimal price path with signaling can also be increasing if consumers learn about quality or their idiosyncratic tastes from repeat purchases, as in Milgrom and Roberts (1986) and Judd and Riordan (1994). In such cases, the seller may use introductory offers to induce learning and repeat purchases. We focus on a setting without repeat purchases.

2.1 Environment

Preferences. Agents attempt to learn the commonly-valued quality of a good, denoted by $\omega \in \mathbb{R}$, based on others' purchase decisions. Each individual i's total valuation for the good derives from both the common value, ω, and a private value (or "taste"), denoted by $t_i \in \mathcal{T} \equiv [\underline{t}, \overline{t}] \subseteq \mathbb{R}$. For simplicity, we assume individual i's total valuation is $u(\omega, t_i) = \omega + t_i$; we discuss at various points how our results extend to more general utility functions. Adopting the good at price p yields a payoff of $u(\omega, t) - p$, while rejecting it yields a payoff normalized to zero. We allow $\mathcal{T}$ to include values such that some types may have a negative valuation for the good; this lets us show how projection may lead to inefficient adoption.

Private values are i.i.d. across individuals with a CDF $F : \mathcal{T} \to [0, 1]$. We assume that F admits a smooth, positive log-concave density $f \equiv F'$. In our formulation of taste projection detailed below, we assume each agent has a misspecified model of F, treating it as excessively concentrated around his own taste relative to the true distribution.

Actions and Timing. A sequence of consumers decide whether to buy the good. In each period $n \in \{1, ..., N\}$, a unit mass of new agents with tastes independently drawn from F enters the market. They simultaneous choose once-and-for-all whether to buy at price $p_n \geq 0$ and then exit. These choices maximize each agent's expected utility given their subjective beliefs over ω. Let d_n denote the resulting quantity demanded in period n.

Information Structure. Agents begin with a non-degenerate common prior over ω. In the main text, we focus on a simple signal structure to make the effects of projection transparent: there is a single signal in the economy and, in each period, a fraction $\lambda \in (0, 1)$ of agents observe its realization, $s \in \mathbb{R}$. Let $\bar{\omega}(s) \equiv \mathbb{E}[\omega|s]$ denote the rational expectation of ω conditional on s. The signal is drawn from a continuous CDF $G(\cdot|\omega)$ that obeys the (strict) Monotone Likelihood Ratio Property in ω so that $\bar{\omega}(s)$ is strictly increasing in s—informed agents will thus take actions based on $\bar{\omega}(s)$ and uninformed agents try to infer $\bar{\omega}(s)$ from these actions. We assume that $\bar{\omega}(s)$ has full support on $\mathbb{R}$; this simplifies the analysis by guaranteeing that projectors will draw a coherent inference from any possible market outcome (i.e., they never observe outcomes they deem impossible). This signal structure is consistent, for instance, with the familiar Gaussian model where the signal and prior are both normally distributed.[8] Moreover, while this simple structure is sufficient to study several features of misinference arising from taste projection, Appendix A shows that the main effects of projection on beliefs continue to emerge in richer structures with heterogeneous signals.[9]

[8]We assume individuals have correct perceptions of the signal structure in order to isolate the effects of taste projection from other biases. In particular, individuals project tastes but not information. Taste projection will, however, distort an individual's perception of others' information.

[9]Appendix A considers two cases: (i) "fully heterogeneous signals," where each agent observes a private independent signal; (ii) "heterogeneous signals across periods," where all agents acting within each period n observe a common signal that is unobserved by agents acting in other periods.

Social Learning. Each Generation $n \geq 2$ observes the pair of price and quantity demanded from the previous generation, (p_{n-1}, d_{n-1}), and use this data to infer their predecessors' beliefs over ω. Since we assume $\bar{\omega}(s)$ has full support on $\mathbb{R}$, any observed pair (p, d) is uniquely rationalized by a feasible value of $\bar{\omega}(s)$ whenever $d \in (0, 1)$, although the value that rationalizes the data will differ across projectors with differing misspecified models. Moreover, as we describe in our specific applications, the fact that a continuum of consumers act in each period implies that the behavior of a preceding generation perfectly reveals $\bar{\omega}(s)$ in the rational equilibrium.[10] Correct social learning is therefore immediate in the rational benchmark of our setup. Taste-projecting agents will nevertheless mislearn: since they have misspecified models, they will incorrectly extract the signal.[11]

Prices. We consider two cases regarding the origin of prices. First, we sometimes consider exogenously determined prices (e.g., a price-taking seller) and describe beliefs as a function of those fixed prices. Second, we consider a profit-maximizing monopolist setting a price p_n at the start of each period n; for this case, we assume the seller has a constant marginal cost normalized to zero and, importantly, is aware of consumers' projection bias. Additionally, the seller observes s prior to period 1 but does not have any private information about ω beyond that of the informed buyers. Since in the settings we consider rational uninformed agents can fully extract s from their predecessors' actions, this assumption guarantees that the seller and rational agents effectively have symmetric information. In this way, we neutralize classical motives for the seller to use prices as signals about ω, which allows us to isolate pricing dynamics that arise entirely due to taste projection.[12]

As such, our focus is on agents drawing inference from demand rather than prices per se. Agents in period n ask themselves what signal s best predicts quantity demanded d_{n-1} when the price is p_{n-1}, but do not attempt to draw any inference about s based on the seller's particular choice of price. While this assumption is admittedly strong, it helps simplify and focus our analysis.[13] Yet, this assumption does not imply that consumers ignore prices when drawing inference. Indeed, the

[10] This environment is somewhat similar to models of sequential observational learning with common preferences in which a single agent acts in each period and takes a continuous action (e.g., Lee, 1993; Eyster and Rabin, 2010). In the rational equilibrium of these models, an agent can perfectly deduce a predecessors' beliefs based on their action. In our setup, an individual agent's action does not reveal their information in the rational equilibrium, but the *aggregate* behavior of agents acting in a single period does reveal their collective information.

[11] Although we assume agents observer the market outcome from only the previous period, this data is sufficient for rational agents to learn the signal. If biased agents were to observe the complete history of outcomes, they may see data inconsistent with their misspecified model. This is because we assume there is a single signal in the market and no other source of noise. We could alternatively avoid such inconsistencies by adding further sources of noise (e.g., prices or demand subject to shocks), but this would significantly complicate the analysis while adding limited additional insight.

[12] As we discuss further below, uninformed agents who do not directly observe s think they can perfectly extract s form the market outcome they observe, regardless of the seller's chosen price. Thus, although the seller and some buyers might have asymmetric information ex ante, buyers expect symmetric information at the interim stage (i.e., when making their choices). This expectation is correct for rational buyers. And projecting buyers who misinfer s still think—albeit wrongly—that they share common information with the seller at the interim stage.

[13] We are not unique in this approach. As discussed above, most existing papers on pricing in markets with observational learning either abstract from cases in which the seller uses prices to signal private information or impose other simplifying assumptions.

price is essential for *interpreting* observed demand—conditional on s, observers rightfully expect fewer sales when p_{n-1} is higher. Put differently, agents in our model infer from others' *reaction* to prices, rather than the chosen price itself. Additionally, the environment we consider is conducive to this assumption since agents believe that d_{n-1} alone is sufficient to reveal s once they know p_{n-1}, regardless of why p_{n-1} was chosen.[14] And perhaps most importantly, we suspect the basic effects of projection on beliefs that we analyze would continue to hold if projectors drew inferences solely based on the realized price; we demonstrate this in Appendix B for a specific case.

2.2 Taste Projection

Gagnon-Bartsch et al. (2021a) provide a general model of taste projection that is applicable to a wide range of Bayesian games. Here, we adapt that model to our particular inferential context.[15]

Channeling Loewenstein et al.'s (2003) model of intrapersonal projection bias, Gagnon-Bartsch et al. (2021a) assume that each agent i perceives the private value of another agent j as a convex combination between j's true value and i's own value; that is, i believes j's private value is $\hat{t}_j(t_i) \equiv \alpha t_i + (1-\alpha)t_j$, with $\alpha \in [0,1)$. The parameter α captures the "degree of projection": $\alpha = 0$ is the rational benchmark, while $\alpha \to 1$ represents the extreme case where an agent believes that others share his exact taste. For tractability, we assume the degree of projection is identical across agents.

Perceptions of the Taste Distribution. The convex-combination specification above implies that agent i's perception of others' private values is described by the random variable

$$\widehat{T}(t_i) \equiv \alpha t_i + (1-\alpha)T, \tag{1}$$

where $T \sim F$ is the true random variable describing private values. Hence, each agent i perceives a distribution of tastes that, relative to reality, is overly concentrated around his own taste, t_i.[16]

This formulation of projection pins down the perceived distributions held by projecting agents, $\{\widehat{F}(\cdot|t)\}_{t\in\mathcal{T}}$, in terms of the true distribution, F, and the projection parameter, α. Each agent perceives a distribution with the same shape as F, but with the probability mass compressed around his own value. The support of this distribution is also compressed when $\mathcal{T}$ is bounded: Equation (1) implies that an agent with type t has a perceived support of $\widehat{\mathcal{T}}(t) \equiv [\underline{t}(t), \bar{t}(t)] \subset \mathcal{T}$, where

[14] As emphasized by Gagnon-Bartsch et al. (2021b), subjectively rational inattention (with respect to a misspecified model) may lead an agent to attend to data he deems sufficient for updating his beliefs (e.g., current demand at a given price), while forgoing careful attention to additional data (e.g., pricing strategy). Indeed, when the seller's cost is subject to noise that is unobserved by buyers, our results are "attentionally stable" in the sense of Gagnon-Bartsch et al. (2021b); that is, a consumer need not confront data that reveals his model's misspecification.

[15] For a review of the motivating evidence, see Gagnon-Bartsch et al. (2021a) and the references therein.

[16] Gagnon-Bartsch et al. (2021a) discusses how this approach naturally extends to cases where players are not symmetric—and thus values are not identically distributed—and to cases where values are correlated. Since we focus on settings with i.i.d. types, we forgo these elaborations.

$\underline{t}(t) \equiv \alpha t + (1-\alpha)\underline{t}$ and $\bar{t}(t) \equiv \alpha t + (1-\alpha)\bar{t}$.[17] Moreover, this type's perceived CDF is

$$\widehat{F}(x|t) = \Pr\left(\widehat{T}(t) \leq x\right) = \begin{cases} 0 & \text{if } x < \underline{t}(t) \\ F\left(\frac{x-\alpha t}{1-\alpha}\right) & \text{if } x \in [\underline{t}(t), \bar{t}(t)] \\ 1 & \text{if } x > \bar{t}(t). \end{cases} \tag{2}$$

These perceived distributions inherit our assumptions on F: each $\widehat{F}(\cdot|t)$ admits a smooth, positive log-concave density $\hat{f}(x|t) = \left(\frac{1}{1-\alpha}\right) f\left(\frac{x-\alpha t}{1-\alpha}\right)$ for $x \in \widehat{\mathcal{T}}(t)$. Intuitively, under these biased perceptions, a projecting agent thinks the average taste is closer to his own than it really is, and underestimates the variance in tastes.

Higher-Order Beliefs. We assume each projector is *naive* about his bias: he neglects that he and others mispredict the distribution of tastes and therefore fails to appreciate that others form discrepant perceptions of this distribution. An agent with private value t thus believes that (i) all others think that private values are distributed according to $\widehat{F}(\cdot|t)$, and (ii) this mutual perception is common knowledge. In essence, people imagine they are playing a game with common knowledge of the environment when in fact perceptions are heterogeneous across players. Our naivete assumption is motivated by the idea that people who are ignorant about their own projection bias are likely not carefully attending to others' projection bias.[18] Naivete also differentiates our model from rational models in which an agent's own taste shapes his beliefs about others' tastes; e.g., correlated private values or uncertainty about F.[19]

Solution Concept. We apply Gagnon-Bartsch et al.'s (2021a) "Naive Bayesian Equilibrum" concept to our setting. Aside from misperceptions about F (and about others' misperceptions of F), we assume projecting agents are otherwise rational and believe others are rational. Each player maximizes his expected payoff according to his distorted beliefs and the presumption that others share his misspecified model. Therefore, each player i plays a BNE strategy of the "perceived game" in which $\widehat{F}(\cdot|t_i)$ is indeed the commonly-known taste distribution. The resulting profile of strategies is a *Naive Bayesian Equilibrium (NBE)*.

To formalize this concept within our setting, suppose the true symmetric game under considera-

[17] Our qualitative results do not hinge on misperceptions of the support per se. They would continue to hold if each type's perceived distribution were approximately the same as Equation (1) but modified to assign a small yet positive probability to types in $\mathcal{T} \setminus \widehat{\mathcal{T}}(t_i)$.

[18] Although studies on the false-consensus effect rarely elicit second-order beliefs, the few that do, e.g. Egan et al. (2014), find that people greatly overestimate how many share their second-order beliefs, which suggests naivete. Of course, our assumption of complete naivete is likely an oversimplification; in the domain of information projection, Danz et al. (2018) find evidence of partial naivete.

[19] Dawes (1989, 1990) argues that the type-dependent beliefs found in experimental studies may reflect rational uncertainty about others' tastes rather than projection bias. However, studies responding to this critique find that subjects' perceptions systematically overweight their own preference relative to information about others' preferences when making predictions about others (e.g., Krueger and Clement, 1994). Engelmann and Strobel (2012) and Ambuehl et al. (2021) similarly find that a false-consensus bias remains despite access to information about others' choices. Preference misperceptions therefore appear robust even when rational explanations due to limited information are tenuous.

tion is Γ with an action space $\mathcal{A} \subseteq \mathbb{R}$. Let $\Gamma(\widehat{F})$ denote that same game when the type distribution is $\widehat{F}$ instead of F; all other elements of $\Gamma(\widehat{F})$ are identical to Γ. A player with type t thinks the game is $\Gamma(\widehat{F}(\cdot|t))$ and presumes that players will follow a BNE of $\Gamma(\widehat{F}(\cdot|t))$. Let $\tilde{\sigma}(\cdot|t)$ denote a symmetric pure strategy profile within the perceived game $\Gamma(\widehat{F}(\cdot|t))$.

Definition 1. *A symmetric strategy profile $\hat{\sigma} : \mathcal{T} \to \mathcal{A}$ is a symmetric Naive Bayesian Equilibrium (NBE) of Γ if, for all $t \in \mathcal{T}$, there exists a symmetric strategy profile $\tilde{\sigma}(\cdot|t) : \widehat{\mathcal{T}}(t) \to \mathcal{A}$ that is a BNE of $\Gamma(\widehat{F}(\cdot|t))$ and $\hat{\sigma}(t) = \tilde{\sigma}(t|t)$.*

To provide some intuition, each player with taste t introspects about others' behavior within his perceived game, and this process leads him to a conjectured BNE strategy profile, $\tilde{\sigma}(\cdot|t)$, of that game.[20] He then follows the strategy prescribed by this conjectured equilibrium; i.e., he takes action $\tilde{\sigma}(t|t)$. A NBE is the strategy profile that emerges when each player engages in this reasoning.

In our setting, new agents enter each period and decide whether to buy after inferring from their predecessors' choices. We assume that players form these inferences according to a NBE: each observer with taste t thinks the sequence of generations is playing a BNE in which $\widehat{F}(\cdot|t)$ is common knowledge. Thus, any player i in Generation $n \geq 2$ thinks that each player in any previous Generation $k < n$ took the action that maximized her expected utility, where that expectation was with respect to i's erroneous model (due to naivete). Player i consequently thinks that the behavior he observes, d_{n-1}, represents the aggregate behavior of a generation with tastes distributed according to $\widehat{F}(\cdot|t_i)$ that is best responding to the beliefs they formed using i's model.

Note that a BNE strategy in this setting is just a map from a buyer's type, t, and expectation of quality, $\hat{\omega}$, to a binary purchase decision. A projecting buyer correctly understands another buyer's strategy conditional on their type and expectation. However, the aggregate behavior that the projecting buyer observes will depend on the distribution of t and $\hat{\omega}$ in the market. He thus misinterprets aggregate behavior due to two mistakes about these distributions: (i) he misperceives the distribution of types, t, acting in the market; and (ii) he mispredicts others' quality expectations, $\hat{\omega}$, since he neglects that those with different types employ inferential strategies different from his.

3 Steady-State Equilibrium with Fixed Pricing

We begin by showing how taste projection distorts beliefs in a static model, which can be interpreted as the steady-state equilibrium of our dynamic model (in Section 4) when the price is held constant across periods. This analysis allows us to establish a few key implications of mislearning due to taste projection before moving to the more complex dynamic setting; it also demonstrates that the

[20] Because $\widehat{F}(\cdot|t)$ inherits our assumptions on F, existence of such a BNE in the perceived game $\Gamma(\widehat{F}(\cdot|t))$ follows from the existence of a BNE in the original game Γ.

comparative statics that arise in the dynamic context robustly emerge in the steady-state as well. Namely, an agent's perceived quality is (i) decreasing in his private taste, and (ii) increasing in the price. As a further implication, the perceived total valuations of agents in equilibrium are excessively similar to one another, leading to a market demand that overreacts to price changes.

The setup mirrors the environment from Section 2.1. A continuum of potential buyers with unit mass face a fixed price p. A fraction $\lambda \in (0, 1)$ of the agents privately observe the realization of $S \sim G(\cdot|\omega)$ and the remaining fraction $1 - \lambda$ do not. The "uninformed agents"—those who do not observe the signal—attempt to extract this information from the equilibrium level of demand.

The steady-state equilibrium follows a logic similar to a rational-expectations equilibrium (e.g., Grossman, 1976; Grossman and Stiglitz, 1980), except agents wrongly use their misspecified models to extract signals. Suppose the fraction of agents who buy is $d \in [0, 1]$. Each uninformed agent follows an inference rule that maps d into an expectation over ω, and then buys the good if their expected valuation exceeds p. In equilibrium, agents' inferences about ω must be consistent with the observed quantity demanded, and this quantity must in turn be consistent with agents' inferences.

We now derive the equilibrium. Informed agents base their buying decisions entirely on s, as they know there is nothing more to learn. Thus, an informed agent with taste t buys if $\bar{\omega}(s) + t \geq p$, and the demand among informed agents is $D^I(p; \bar{\omega}(s)) \equiv \Pr\left[\bar{\omega}(s) + T \geq p\right] = 1 - F\left(p - \bar{\omega}(s)\right)$. Reflecting our interest in states where consumers should rationally take heterogeneous actions, we say that the pair (p, s) *admits interior demand* when $D^I(p; \bar{\omega}(s)) \in (0, 1)$.

Uninformed agents infer $\bar{\omega}(s)$ from the aggregate quantity demanded, d. To build intuition, we first describe agents' inferences in the rational benchmark. Let $\hat{\omega}(d)$ denote the inferred value of $\bar{\omega}(s)$ upon observing d. Demand among the uninformed is thus $\Pr[\hat{\omega}(d) + T \geq p] = 1 - F(p - \hat{\omega}(d))$, and the total demand is

$$d = \lambda \cdot \underbrace{\big(1 - F\big(p - \bar{\omega}(s)\big)\big)}_{\text{Demand among the informed}} + (1 - \lambda) \cdot \underbrace{\big(1 - F\big(p - \hat{\omega}(d)\big)\big)}_{\text{Demand among the uninformed}} . \tag{3}$$

We require that $\hat{\omega}(d)$ is Bayes-rational given an agent's model. Hence, in the rational benchmark—where players share common knowledge of F—the unique symmetric inference rule is $\hat{\omega}(d) = p - F^{-1}(1 - d)$. When following this rule, the observed quantity demanded d is such that uninformed agents infer $\hat{\omega}(d) = \bar{\omega}(s)$ and hence behave as informed agents. This follows from the fact that, in equilibrium, d reveals the marginal type.[21]

This strategy of identifying others' information off of the inferred marginal type leads projectors astray since they misinfer the valuation of the marginal buyer. More specifically, a projecting agent

[21]For instance, if 30% of the market buys at p, then the marginal buyer has a private value at the 70th percentile of F. Thus, rational uninformed agents who anticipate d simply choose to buy if their taste is above the 70th percentile and decline otherwise.

thinks the market is in the rational equilibrium described above, and draws inferences following that logic. They do so, however, using their misspecificed model. A buyer with taste t_i thinks the demand function among informed agents is $\widehat{D}^I(p;\bar{\omega}(s)|t_i) \equiv 1 - \widehat{F}\big(p - \bar{\omega}(s)|t_i\big)$. Furthermore, due to naivete, he thinks others share his perception of F—and hence of the demand function—and thus he thinks others will draw the same inference as him from d. Thus, an agent with taste t_i thinks the rational symmetric inference rule is $\hat{\omega}(d|t_i)$ and that, in equilibrium, $\hat{\omega}(d|t_i)$ must satisfy

$$d = \lambda \cdot \underbrace{\Big(1 - \widehat{F}\big(p - \bar{\omega}(s)|t_i\big)\Big)}_{\text{Perceived demand among the informed}} + (1-\lambda) \cdot \underbrace{\Big(1 - \widehat{F}\big(p - \hat{\omega}(d|t_i)|t_i\big)\Big)}_{\text{Perceived demand among the uninformed}} . \tag{4}$$

Consequently, an agent with taste t_i comes to believe that the value of $\bar{\omega}(s)$ is

$$\hat{\omega}(d|t_i) = p - \widehat{F}^{-1}(1-d|t_i). \tag{5}$$

This inferential strategy would correctly extract others' information if agent i's misspecified model were correct (i.e., if $T \sim \widehat{F}(\cdot|t_i)$ and agents shared this belief).[22]

The misinference described above involves two distinct errors. One stems from an error in first-order beliefs: agent i's conjectured equilibrium condition (Equation 4) wrongly posits that tastes are distributed according to $\widehat{F}(\cdot|t_i)$ instead of F. Additionally, due to naivete, agent i's erroneous second-order beliefs cause him to think others draw the same inference as him, $\hat{\omega}(d|t_i)$, since he neglects that others employ discrepant models.

In truth, the demand among uninformed agents arises from each type of agent acting on their distinct equilibrium inference. The equilibrium quantity demanded is then the value of d solving

$$d = \lambda \cdot D^I\left(p;\bar{\omega}(s)\right) + (1-\lambda) \cdot \underbrace{\Pr[\hat{\omega}(d|T) + T \geq p]}_{\text{Demand from Uninformed Agents}} , \tag{6}$$

where $\hat{\omega}(d|t)$ is given by (5) for each $t \in \mathcal{T}$. This equilibrium quantity, call it d^*, pins down the profile of agents' perceptions of $\bar{\omega}(s)$. We denote this profile by $\hat{\omega}(t)$; that is, $\hat{\omega}(t) = \hat{\omega}(d^*|t)$. The following proposition establishes two central properties of misinference under taste projection.

Proposition 1. *Suppose (p, s) admits interior demand. For any $\alpha > 0$, there exists a unique equilibrium profile of beliefs, and it has the following properties:*

1. *$\hat{\omega}(t)$ is strictly decreasing in t. Moreover, there exists an interior type $\tilde{t}$ such that agents with $t > \tilde{t}$ underestimate ω while those with $t < \tilde{t}$ overestimate ω.*

2. *For each type $t \in \mathcal{T}$, the perception $\hat{\omega}(t)$ is strictly increasing in p.*

[22] Note that $\widehat{F}^{-1}(1-d|t_i) \to F^{-1}(1-d)$ for all t_i as $\alpha \to 0$. Hence, each agent's inference collapses to the common rational inference as projection vanishes.

Part 1 of Proposition 1 establishes that quality perceptions are inversely related to tastes. If agent i has a high private taste, he expects that others do too, exaggerating the fraction of people who buy conditional on price p and belief $\bar{\omega}(s)$. Accordingly, the actual demand at price p looks rather weak and, to rationalize it, he must infer a relatively low common value. Conversely, if agent i has a low private taste, he will infer a relatively high common value. In other words, the interpretation of a good's popularity is in the eye of the beholder.

Where is the divide between types who overestimate quality and those who underestimate it? As noted above, inference in this setting stems from identifying the valuation of the marginal consumer. The nature of projectors' misinference can thus be understood from how they misidentify the marginal type. Suppose that in equilibrium a fraction z of consumers turn down the good. The marginal type thus has a private value t^* at the z^{th} percentile of the taste distribution. An uninformed consumer tries to deduce t^* since this would reveal $\bar{\omega}(s)$ via the indifference condition $t^* = p - \bar{\omega}(s)$. However, a projector misperceives the private value at each percentile other than his own. To see this, let $\hat{t}(z|t_i)$ be the perceived type at the z^{th} percentile according to an agent with taste t_i, and let $t^*(z)$ denote the true type. From (2), $\hat{t}(z|t_i)$ value solves

$$z = \widehat{F}(\hat{t}(z|t_i)|t_i) = F\left(\frac{\hat{t}(z|t_i) - \alpha t_i}{1-\alpha}\right) \quad \Rightarrow \quad \hat{t}(z|t_i) = \alpha t_i + (1-\alpha)t^*(z). \tag{7}$$

Reflecting the idea that projectors think others' values are compressed around their own, type t_i's perception of the type at the z^{th} percentile is shifted toward his own. This recasts the intuition from above: those with high private values overestimate the marginal type, and thus underestimate the good's quality; those with low private values do the opposite. Furthermore, this means that a projector who is at the z^{th} percentile himself—who has a taste matching that of the informed marginal type—is the unique type who infers $\bar{\omega}(s)$ correctly. To summarize: (i) $\hat{\omega}(t^*) = \bar{\omega}(s)$ where $t^* = p - \bar{\omega}(s)$ is the rational marginal type; (ii) $\hat{\omega}(t) < \bar{\omega}(s)$ for all agents with $t > t^*$; and (iii) $\hat{\omega}(t) > \bar{\omega}(s)$ for all agents with $t < t^*$.

Part 2 of Proposition 1 shows that, irrespective of their private taste, projecting agents form higher perceptions of the common value when p is higher. This stems from the fact that projectors underestimate the heterogeneity in others' private values and, therefore, underestimate the fraction of types who would remain in the market at a higher price. If the price were to increase, a projector would see more remain than he expected; to rationalize this discrepancy, he must then infer a higher quality. Figure 1 depicts this intuition. First, note that a projector's inferred quality $\hat{\omega}$ is such that his perceived demand function given $\hat{\omega}$, $\widehat{D}(\cdot;\hat{\omega}|t)$, passes through the market outcome, (d, p). As the price increases from p' to p'', the quantity demanded adjusts along the true demand curve, $D(\cdot;\bar{\omega}(s))$. The new quantity, however, is inconsistent with the projectors' demand curve that rationalized the outcome at p': since a projector underestimates heterogeneity, their perceived demand curve is a

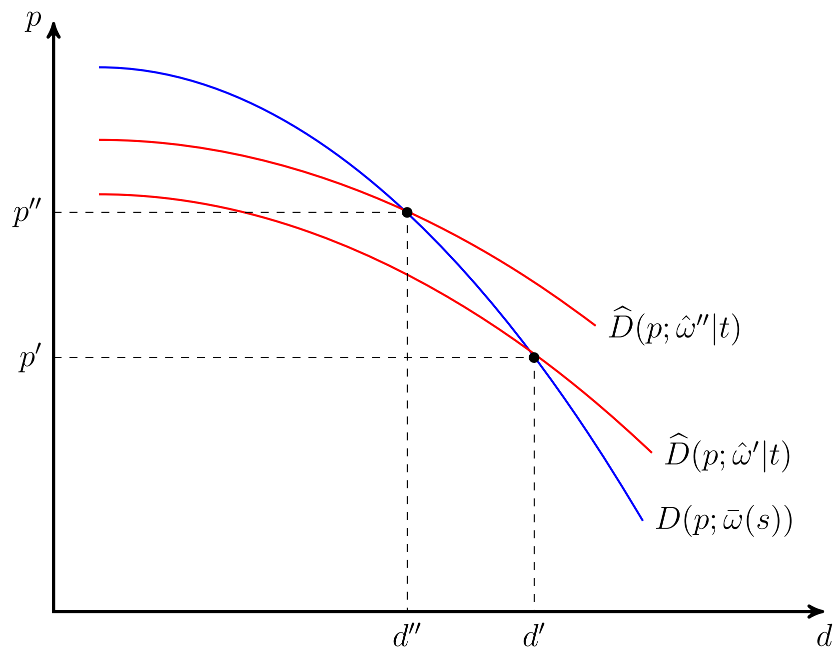


Figure 1: *True and Perceived Equilibrium Demand Functions.*

counter-clockwise rotation of $D(\cdot;\bar{\omega}(s))$ (see Johnson and Myatt, 2006) and is thus more price elastic. Hence, to rationalize the quantity demanded at price p'', the projector will form a higher expectation of ω, consistent with an outward shift of his perceived demand curve.

Another intuition for this result reflects the discussion above about identifying the marginal type. The farther a type is from the margin, the more distorted is his perception of the marginal type. Thus, a high type who is above the margin at price p will be closer to the margin after a small price increase. Since this high type originally underestimates ω, he will underestimate ω by less if the price increases. A similar logic holds for those below the margin at price p: they will be farther from the margin after a price increase, and hence they will subsequently overestimate ω by more. In other words, the higher is the true marginal type, the higher is each projector's perception of ω.

While the results of Proposition 1 hold more generally, they are particularly transparent when $u(\omega, t) = \omega + t$.[23] In this case,

$$\hat{\omega}(t) = (1-\alpha)\bar{\omega}(s) + \alpha(p - t). \tag{8}$$

The degree of projection, α, drives both the positive distortionary effect of p and the negative distortionary effect of an individual's taste. Furthermore, an uninformed agent's perceived *total* value of the good is $\hat{\omega}(t) + t = (1-\alpha)(\bar{\omega}(s) + t) + \alpha p$. Thus, as α increases, a projector's idiosyncratic taste t has less influence on their perceived valuation. Importantly, this implies that the perceived

[23] Our proofs of Propositions 1 and 2 in Appendix C establish these results for more general utility functions. Moreover, these results hold for additional signal structures as well, as we discuss in Appendix A.

values among uninformed agents exhibit less variation than they would under rational inference.

Proposition 2. *Suppose (p, s) admits interior demand. For any $\alpha > 0$, the (mis)perceived valuations of agents in the steady-state have diminished variance relative to the rational benchmark.*

Proposition 2 reveals a sense in which taste projection is self-fulfilling: when agents initially believe that idiosyncratic tastes are more similar than they really are, their distorted inferences lead to perceived valuations that are, in fact, more similar than they ought to be. In other words, the agents' initial misperception of the environment generates data that confirms their misperception.[24]

Given the distortions in beliefs described above, what price would maximize a seller's profits? In this particular setting, the optimal static price under projection is the same as under rational learning. This is because taste projection does not distort the quantity demanded: in equilibrium, the same set of consumers adopt the good regardless of whether they are rational or suffer from projection. As noted above, this happens because the buyer with a type matching that of the rational marginal type learns correctly and is therefore still marginal under projection. However, the fact that projection affects beliefs but not market outcomes here is an artifact of the particular setting—namely, because the price is fixed and consumers have unit demand. Although this setting is ideal for developing intuitions on why and how projection distorts beliefs, the following sections show how relaxing these features will cause biased beliefs to directly influence market outcomes.

In particular, the next section examines how the seller could benefit from dynamic pricing. Proposition 2 additionally implies that demand among misinformed consumers will overreact in the short run to a change in price. That is, if consumers use their perceptions of $\hat{\omega}(t)$ formed in an equilibrium with price p to decide whether they should buy at a new price p', then the demand response to this price change will exceed the rational benchmark. For an intuition, recall that projecting consumers who are above the margin at price p underestimate ω. Thus, relative to rational consumers, they are less willing to continue buying after a price increase. Similarly, projecting consumers who are initially below the margin overestimate ω, and thus they are too eager to buy after a price reduction. Dynamic pricing allows the seller to take advantage of these effects.

4 Dynamic Pricing

We now turn to the dynamic setup introduced in Section 2. Section 4.1 first characterizes how beliefs evolve under an arbitrary price path. Section 4.2 then analyzes dynamic monopoly pricing.

[24] This result also suggests caution when measuring heterogeneity in consumers' preferences since the misperceived valuations among projectors will exhibit less heterogeneity than their true values. Furthermore, measurements of taste projection in markets should also account for this endogeneity problem: while it may appear that there is low variance in valuations *and* that consumers think there is low variance, the former may be caused by the latter via biased learning.

In each period $n = 1, 2, \ldots, N$, a unit mass of new consumers with tastes independently drawn from F enters the market. Each consumer in Generation n makes a once-and-for-all decision whether to adopt the good. In each generation $n \geq 2$, (i) all individuals observe the price and aggregate demand from the previous generation, (p_{n-1}, d_{n-1}), and (ii) a fraction $\lambda \in (0,1)$ privately observe s. Thus, $1-\lambda$ uninformed consumers in each generation $n \geq 2$ engage in social learning while the informed consumers simply follow the signal.

In period 1, consumers must make decisions based solely on their private information. To simplify matters, we assume all consumers in period 1 observe s. There are two interpretations of this assumption: (i) early consumers have greater access to information than later ones (e.g., initial advertising or "hands-on" promotions spread information more widely early on); (ii) the market begins in the steady-state equilibrium derived in Section 3. Under the second interpretation, our results here describe the short-run dynamics of beliefs and behavior when price changes move the market out of the steady state. This assumption also simplifies the analysis by ensuring that the seller does not have an informational advantage over buyers, thereby neutralizing any incentive for the seller to use prices to signal quality (see the discussion at the end of Section 2.1).[25]

Rational learning is straightforward. Since a continuum of agents act in each period, the aggregate demand from the previous period perfectly reveals the signal when there is common knowledge of F (and of rationality). While agents learn immediately in the rational benchmark, projectors do not since they wrongly extract the signal as if it were common knowledge that $T \sim \widehat{F}(\cdot|t_i)$.

4.1 The Dynamics of Biased Beliefs and the Effect of Prices

We first describe how beliefs evolve under an arbitrary price path. We begin by characterizing the beliefs and behavior of uninformed consumers in period 2 upon observing (p_1, d_1).

In period 1, aggregate demand is equal to the rational benchmark: $d_1 = D^I(p_1; \bar{\omega}(s)) = 1 - F(p_1 - \bar{\omega}(s))$.[26] In period 2, an individual with taste t thinks that when buyers in period 1 have expectations equal to $\hat{\omega}$, their demand is

$$\widehat{D}^I(p_1; \hat{\omega}|t) = 1 - \widehat{F}(p_1 - \hat{\omega}|t) = 1 - F\left(\frac{p_1 - \hat{\omega} - \alpha t}{1-\alpha}\right). \tag{9}$$

This buyer will then infer a value of $\hat{\omega}$ that solves $\widehat{D}(p_1; \hat{\omega}|t) = d_1$. Denoting this value by $\hat{\omega}_2(t)$,

[25] While the assumption that all consumers in Generation 1 are privately informed simplifies the analysis in various ways, it does not significantly influence the results. For instance, if a fraction $\lambda < 1$ of consumers observe s in each period $n = 1, 2, \ldots$ and face a fixed price, then the environment corresponds to the dynamic analog of the static model in Section 3: as $n \to \infty$, beliefs and behavior converge to the steady-state values described in Section 3.

[26] This follows from our assumption that all consumers in period 1 are informed (i.e., they observe s).

the previous condition yields

$$\hat{\omega}_2(t) = (1-\alpha)\bar{\omega}(s) + \alpha(p_1 - t). \tag{10}$$

Notice that the misinferences among observers in this dynamic context exhibit the same steady-state properties described in Propositions 1 and 2 from the static case. Indeed, (10) exactly matches the steady-state perceptions derived in Equation (8). These perceptions are decreasing in an observer's taste, increasing in the price, and give rise to perceived total valuations that exhibit too little heterogeneity relative to the rational benchmark.

Building on that final point, we can show that the demand function of uninformed types in period 2 is locally more elastic with respect to p_2 than the rational one (Johnson and Myatt, 2006). More specifically, it is a counter-clockwise rotation of the demand function of informed types, and the rotation point is the market outcome from the previous period, (p_1, d_1). Notice that if we let $\bar{\omega}_2 \equiv (1-\alpha)\bar{\omega}(s) + \alpha p_1$ denote the "taste-independent" (mis)perception of $\bar{\omega}(s)$ among consumers in period 2, then (10) implies that each uninformed consumer i's perceived total valuation is $u\left(\hat{\omega}_2(t_i), t_i\right) = \bar{\omega}_2 + (1-\alpha)t_i$. The demand among uninformed consumers in period 2 is thus

$$D^U(p_2; \bar{\omega}_2) \equiv \Pr\left[u\left(\hat{\omega}_2(T), T\right) \geq p_2\right] = 1 - F\left(\frac{p_2 - \bar{\omega}_2}{1-\alpha}\right). \tag{11}$$

By contrast, under rational inference, this demand would match that of informed consumers; i.e., $D^I(p_2; \bar{\omega}(s)) = 1 - F(p_2 - \bar{\omega}(s))$. It is clear that $\alpha > 0$ implies that $D^U(p_2; \bar{\omega}_2)$ is more sensitive to p_2 than demand among rational observers with those same beliefs (see Figure 2). The rationale builds from intuitions developed in the static case: in period 2, perceptions of $\bar{\omega}(s)$ are declining in consumers' private values, and the buyer with a private value equal to that of the marginal type from period 1, denoted t_1^*, is the unique uninformed type who infers $\bar{\omega}(s)$ correctly. Those with private values above t_1^* see a weaker demand in period 1 than anticipated in state $\bar{\omega}(s)$ and consequently underestimate ω. If $p_2 > p_1$, then only those types with overly pessimistic beliefs will be served in period 2, and the quantity demanded will thus fall below the rational benchmark at p_2. In contrast, those with $t < t_1^*$ see a stronger demand than anticipated in state $\bar{\omega}(s)$ and overestimate ω. If $p_2 < p_1$, then those with overly optimistic beliefs will be served—the marginal type will be among this contingent—and hence the quantity demanded will exceed the rational benchmark.

Now we analyze how beliefs and aggregate behavior evolve over time. Generation 3 forms their quality expectations based on the quantity demanded in period 2, which is

$$d_2 = D(p_2; \bar{\omega}_2; \bar{\omega}(s)) \equiv \lambda D^I(p_2; \bar{\omega}(s)) + (1-\lambda) D^U(p_2; \bar{\omega}_2). \tag{12}$$

While misinference among Generation 2 stemmed directly from misunderstanding others' tastes

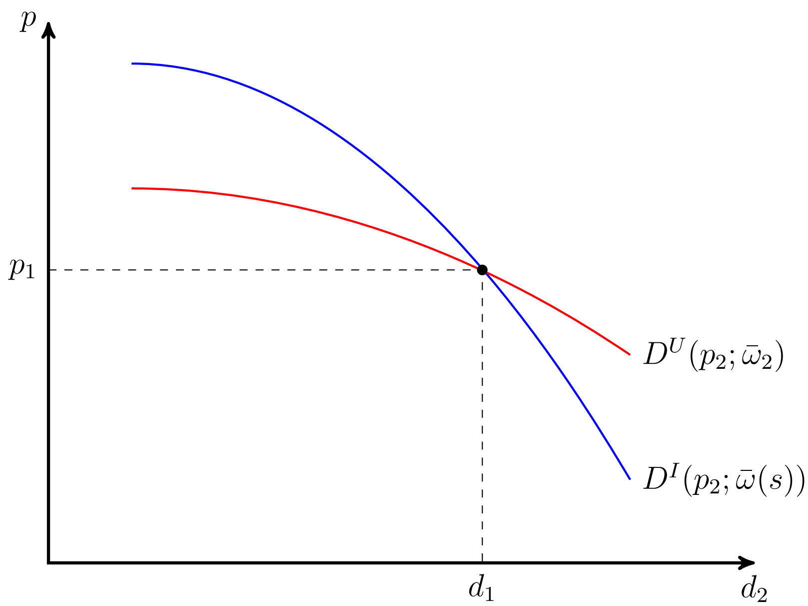


Figure 2: *Demand Functions of the Informed and Uninformed in Period 2.*

(i.e., an error in first-order beliefs), the misinference among Generation 3 also includes a "social misinference" effect stemming from naivete about others' projection. Namely, individuals neglect that their predecessors failed to reach consistent beliefs. Since uninformed consumers expect to extract s form their predecessors' behavior, an individual in period 3 accordingly thinks that the uninformed consumers in period 2 consistently and correctly inferred s and are thus now informed. This presumption is false: projectors in period 2 draw biased, taste-dependent beliefs (as in Equation 10). Nevertheless, an uninformed projector in Generation 3 with taste t thinks period-2 demand is determined by the function $\widehat{D}^I(p_2;\hat{\omega}|t)$ in (9)—she does not realize that it derives from a composition of demand functions as in (12). This observer then infers a value of $\hat{\omega}$ that solves $d_2 = \widehat{D}^I(p_2;\hat{\omega}|t)$, which we denote by $\hat{\omega}_3(t)$. As with Generation 2, if we let $\bar{\omega}_3$ denote the taste-independent part of $\hat{\omega}_3(t)$, then we can write $\hat{\omega}_3(t) = \bar{\omega}_3 - \alpha t$. Aggregate demand among Generation 3 then follows the same form as Generation 2: $d_3 = D(p_3;\bar{\omega}_3,\bar{\omega}(s))$ where D is as defined in (12).

A similar logic unfolds in each period $n \geq 2$. The perceived quality among uninformed agents in Generation n can be written in terms of a taste-independent component, denoted by $\bar{\omega}_n$, which we refer to as the *aggregate biased belief* in period n.

Lemma 1. *In each period $n = 2, \ldots, N$, the expected quality that an uninformed agent with taste t infers is $\hat{\omega}_n(t) = \bar{\omega}_n - \alpha t$, where $\bar{\omega}_n$ is independent of t. Thus, the sequence of aggregate biased beliefs, $(\bar{\omega}_n)$, is a sufficient statistic for each type's belief over time.*

Despite a continuum of types forming distinct beliefs from each observation, Lemma 1 implies that we can account for this infinite-dimensional process by studying the evolution of the unidimensional sequence, $(\bar{\omega}_n)$. Since this sequence describes the path of uninformed consumers' beliefs, the quantity demanded in each period n, d_n, is given by

$$D(p_n;\bar{\omega}_n,\bar{\omega}(s)) = \underbrace{\lambda\big[1-F(p_n-\bar{\omega}(s))\big]}_{\text{Informed Demand}} + \underbrace{(1-\lambda)\left[1-F\left(\frac{p_n-\bar{\omega}_n}{1-\alpha}\right)\right]}_{\text{Uninformed Demand}}. \tag{13}$$

However, an uninformed consumer in period $n+1$ thinks that d_n is determined by

$$\widehat{D}(p_n;\bar{\omega}_{n+1}) \equiv 1-F\left(\frac{p_n-\bar{\omega}_{n+1}}{1-\alpha}\right).$$ [27]

Furthermore, $\bar{\omega}_{n+1}$ must be consistent with d_n for all $n \geq 2$; that is, $d_n = \widehat{D}(p_n;\bar{\omega}_{n+1})$. Hence, the law of motion describing the process $(\bar{\omega}_n)$ is characterized by the condition

$$\widehat{D}(p_n;\bar{\omega}_{n+1}) = D(p_n;\bar{\omega}_n,\bar{\omega}(s)), \tag{14}$$

starting from the initial condition of $\bar{\omega}_2 = (1-\alpha)\bar{\omega}(s)+\alpha p_1$.

Therefore, the way market demand unfolds across periods depends on how the aggregate biased beliefs $(\bar{\omega}_n)$ evolve, which is itself influenced by the entire history of market prices.

Proposition 3. *For any $\alpha > 0$, the aggregate biased belief in period $n \geq 2$, $\bar{\omega}_n$, is strictly increasing in each p_k for $k < n$.*

Proposition 3 shows that in a dynamic setting, the biased perceptions in period n are increasing in all past prices; hence, the manipulating effect of prices on the beliefs of projectors is amplified compared to the static setting of Section 3. In particular, prices in earlier periods have a long-lasting effect on beliefs which, as we will show in the next section, leads a monopolist to adopt a declining-price strategy.

Before turning to the optimal price path given this belief process, we describe outcomes under two benchmark scenarios: (i) a constant price, and (ii) a single change in price. First, if the price is fixed at p (e.g.,. the market is in a competitive equilibrium or other frictions mandate a fixed price), then $\bar{\omega}_n = \bar{\omega}_2$ for all $n > 2$. Beliefs remain constant over time, and the quantity demanded in each period matches the rational benchmark at price p. Intuitively, since the type in Generation 2 who learns correctly has a private value equal to the rational marginal type, this type will again be marginal given that the price is constant. Hence, Generation 2 demands the same quantity as Generation 1. Since Generation 3 then observes the same quantity as Generation 2 did, they draw the same inference. This result reflects the notion that our dynamic process can be viewed as starting from the steady-state: when the price stays constant, the system remains fixed.

[27] More precisely, an uninformed consumer in period $n+1$ with taste t thinks d_n is determined by $\widehat{D}^I(p_n;\hat{\omega}_{n+1}(t)|t)$ as in (9). Applying the fact that $\hat{\omega}_{n+1}(t) = \bar{\omega}_{n+1} - \alpha t$ yields the expression here.

On the other hand, when the price changes, aggregate demand will initially overreact and then slowly converge back to the rational level given the new price. The logic is similar to the reason why demand among the uninformed in Generation 2 is excessively sensitive to p_2 (e.g., the discussion around Figure 2). For instance, suppose the price permanently drops in period 2. All uninformed types with a private value below the marginal type from Generation 1 overestimate ω; hence, relative to the rational benchmark, a larger measure of those who were originally submarginal buy once the price drops. A similar overreaction occurs if the price instead increases.

Proposition 4. *Suppose there exists a period $n^* \geq 1$ such that $p_n = p$ for $n \leq n^*$, and $p_n = \tilde{p} \neq p$ for all $n > n^*$. Consider s such that both (p, s) and $(\tilde{p}, s)$ admit interior demand, and let $\tilde{d}$ denote the quantity demanded at price $\tilde{p}$ under rational learning. Then, for any $\alpha > 0$ the following hold:*

1. Initial Overreaction: *If $\tilde{p} > p$, then $d_n < \tilde{d}$ for all $n > n^*$. If instead $\tilde{p} < p$, then $d_n > \tilde{d}$ for all $n > n^*$.*
2. Convergence to Rational Equilibrium: *$|d_n - \tilde{d}|$ is decreasing in n and $\lim_{n\to\infty} |d_n - \tilde{d}| = 0$.*

Social learning under taste projection therefore offers a novel explanation for temporary overreaction to price changes, thereby complementing other existing—yet conceptually distinct—explanations. For instance, a change in the price could momentarily increase attention or salience to the price shortly thereafter (Bordalo et al., 2013, 2020). Or consumers with a "taste for bargains" may experience additional elation when buying at a price below some reference level (e.g., the previous price), thereby leading more to buy while the new price still feels like a "deal" (Jahedi, 2011; Armstrong and Chen, 2020).

4.2 Optimal Monopoly Pricing

We now analyze how a sophisticated seller optimally sets prices over time when facing taste-projecting consumers. The seller chooses a sequence of prices $(p_1, \ldots, p_N)$ to maximize

$$\Pi \equiv p_1 D^I(p_1; \bar{\omega}(s)) + \sum_{n=2}^{N} p_n D(p_n; \bar{\omega}_n, \bar{\omega}(s)) \tag{15}$$

subject to the dynamic constraint in (14) for all $n \geq 2$.[28] In order for the next generation to draw a well-defined inference from quantity demanded, we require that the seller serves a positive fraction of consumers in each period. We operationalize this by imposing a price ceiling that is arbitrarily close to the valuation of the highest informed type: $\bar{p} \equiv \bar{\omega}(s) + \bar{t} - \kappa$ for some small $\kappa > 0$.[29]

[28] For simplicity, we abstract from the seller discounting future profits. All of our results would continue to hold if the seller exponentially discounted future profits with a discount factor $\delta \in (0, 1)$.

[29] This price ceiling will have little effect on projectors' beliefs and behavior since projectors can never be induced to have a willingness to pay above the highest informed type. The price ceiling is also not consequential for our qualitative

Let p_n^* denote the seller's profit-maximizing price in period n. Under rational learning, all consumers will correctly infer s, and the seller essentially faces an identical market of informed consumers in each period. Let p^M denote the static optimal monopoly price when facing informed consumers. The optimal price path in the rational benchmark (i.e., $\alpha = 0$) is to simply charge $p_n^* = p^M$ for all n. As we emphasize below, this is not so with projecting consumers (i.e., $\alpha > 0$).

Our analysis first considers the two-period case, which will be sufficient for showing how prices influence and respond to projectors' erroneous beliefs. We then consider longer horizons. Unlike in the two-period case, projectors in later rounds form beliefs after observing the *irrational* behavior of projectors who acted previously. While this difference introduces a richer set of incentives for the seller's pricing strategy, we show that the optimal price path still starts high and gradually declines.

4.2.1 Two-Period Model

Taste projection among consumers introduces dynamic pricing incentives for the seller. Indeed, since the current price inflates the beliefs of consumers in later periods, the seller may benefit from increasing today's price—at the cost of losing immediate sales—in order to increase perceptions and demand among future consumers. The benefit from such manipulation is clearly suggested by the distorted beliefs formed in Generation 2, as described in (10). The private value of the marginal type in Generation 1 determines the threshold in the taste distribution where $\hat{\omega}_2(t)$ switches from overestimating quality to underestimating it. As this threshold is increasing in p_1, a higher p_1 will result in a larger share of individuals in Generation 2 who overestimate quality.[30]

But is it worthwhile for the seller to forego sales today in order to boost demand in the future? The answer is unambiguously yes. To provide intuition, consider two pricing strategies: (i) constant pricing, where $p_1 = p_2 = p^M$ and (ii) declining prices such that $p_1 = p^M + \epsilon$ and $p_2 = p^M - \epsilon$ for some $\epsilon > 0$. The first strategy generates profits identical to the rational benchmark. While the second strategy generates diminished sales in period 1 relative to the rational benchmark, it generates a disproportionate expansion in period 2. This happens because the demand curve in Generation 2 is a counter-clockwise rotation around p_1 of the demand curve from the previous generation. Locally, a small reduction of p_2 below p^M leads to a greater expansion in period-2 sales compared to the contraction of period-1 sales induced by a commensurate increase of p_1 above p^M. This follows from the fact that those who were previously submarignal hold inflated perceptions; hence, a price cut attracts an exaggerated share of consumers (as in Proposition 4). As a result, the profits gained in period 2 more than offset those lost in period 1.[31] This intuition holds more generally.

results: the optimal price path still involves an inflated price in period 1 and a subsequent price reduction regardless of whether p_1 is at the ceiling or not. Furthermore, for every value of α, there exists a value $\bar{\lambda}$ such that $\lambda > \bar{\lambda}$ guarantees an interior solution to the seller's problem, rendering the ceiling irrelevant.

[30] Notably, projection induces this dynamic interdependence even in settings, such as ours, where there is no temporal link in pricing in the rational model.

[31] By a similar logic, choosing $p_2 > p_1$ is particularly costly for the seller, as this would exclude optimistic consumers

Proposition 5. *Consider any s such that (p^M, s) admits interior demand.*

1. *For any $\alpha > 0$, we have $p_1^* > p^M$ and $p_1^* > p_2^*$.*

2. *The seller's profit under the optimal price path is increasing in α and decreasing in λ.*

Intuitively, as α increases, there is greater scope to manipulate beliefs, thereby increasing the seller's profit above the rational benchmark. The seller's profit is instead decreasing in λ: with fewer uninformed agents in the market, it becomes more costly to deviate from the rational-benchmark price. Additionally, although p_1^* always exceeds p^M (i.e., the rational-benchmark price), the relationship between p_2^* and p^M depends on the degree of projection. When α is low and projectors' beliefs are only mildly distorted by p_1, the seller optimally chooses $p_2 < p^M$ in order to induce a large share of overoptimistic types to buy. When α is high and beliefs are strongly distorted by p_1, then even a $p_2 > p^M$ can induce these types to buy.

Pricing under projection clearly harms consumers in period 1 since $p_1^* > p^M$. But it also harms some consumers in period 2: beliefs are manipulated in a way that leads some low-valuation consumers to buy at a price they would refuse under rational learning. While some of this harm to consumers' surplus simply represents a transfer to the seller, sufficiently strong projection can induce consumers with truly negative valuations to adopt the good. Such adoption is clearly inefficient.

Proposition 6. *Consider any s such that (p^M, s) admits interior demand.*

1. *For any $\alpha > 0$, under the profit-maximizing price path, there exists a positive measure of types who buy and overpay: for these types, $\bar{\omega}(s) + t < p_2$.*

2. *If there exist types with truly negative valuations, i.e., $\bar{\omega}(s) + \underline{t} < 0$, then there exists a threshold $\tilde{\alpha}$ such that for $\alpha > \tilde{\alpha}$ the profit-maximizing price path induces inefficient adoption: there exists an interval of types t who buy despite $\bar{\omega}(s) + t < 0$.*

An implication of this result is that the seller's optimal pricing scheme always induces excessive take-up among uninformed buyers, consistent with familiar notions of herding or bandwagon effects in markets.[32] Such excessive take-up indeed leads some uninformed consumers to overpay. Figure 3 shows the demand curves among informed (blue) and uninformed consumers (red) in period 2. The demand curve among informed consumers, $D^I(p; \bar{\omega}(s))$, reflects the rational valuation of the marginal buyer for any level of market coverage d. The demand curve among uninformed consumers, $D^U(p; \bar{\omega}_2)$, instead reflects the willingness to pay of the marginal consumer given d. Thus, for any d, the vertical gap between the red and blue curves reflects the wedge between the marginal

while targeting just the pessimistic ones.

[32] It is straightforward to show that the marginal uninformed type in period 2, $\hat{t}_2$, is strictly below the marginal informed type, t_2^*, and the interval of uninformed types who wrongly buy the good has measure $t_2^* - \hat{t}_2 = \frac{\alpha(p_1^* - p_2^*)}{1-\alpha} > 0$.

uninformed consumer's willingness to pay and his true valuation. Manipulative pricing under projection causes a range of uninformed types to buy the good when they should, in fact, abstain given p_2^*: the rational level of demand at p_2^* is d_2^I, yet a market of projectors would demand a quantity $d_2 > d_2^I$. Projectors' consumer surplus is no longer simply the area below their demand curve and above the price, since all consumption beyond d_2^I involves overpaying. Instead, projectors' surplus is the area above p_2^* yet below their valuation curve (area in blue) *minus* the area below p_2^* yet above their valuation curve (area in red). Moreover, the dark red triangle displays a case where such over-adoption is inefficient since consumers with negative valuations are lured into buying.

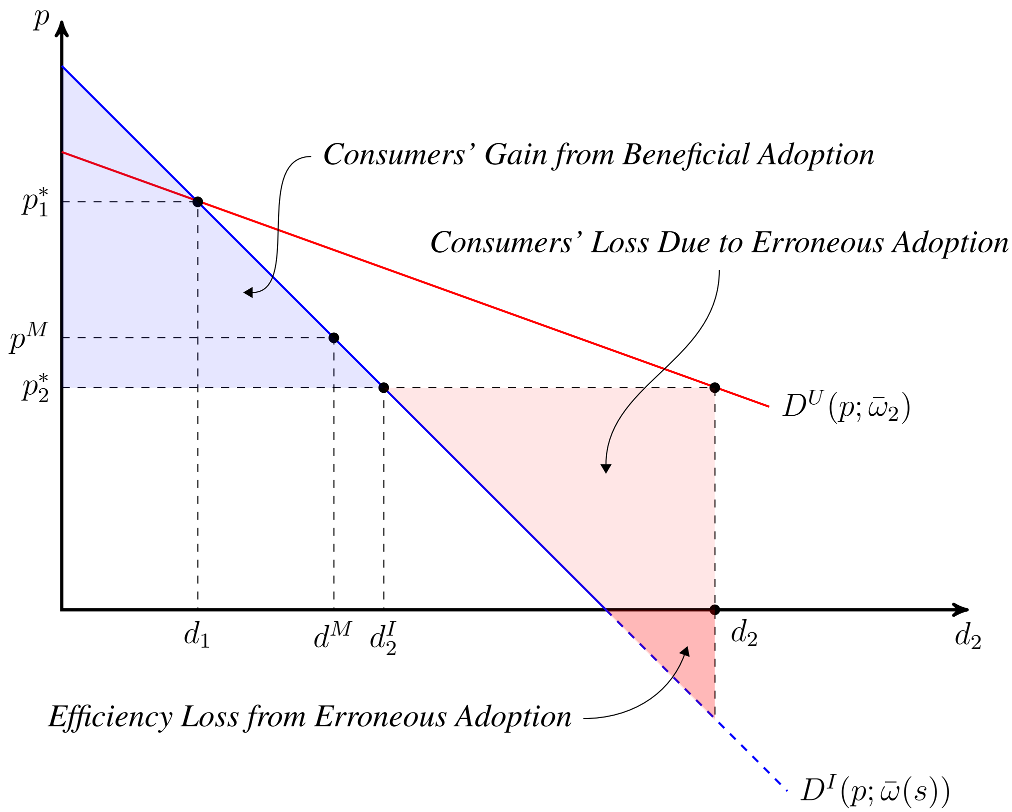


Figure 3: *Demand functions in Period 2 (for both informed and uninformed agents).*

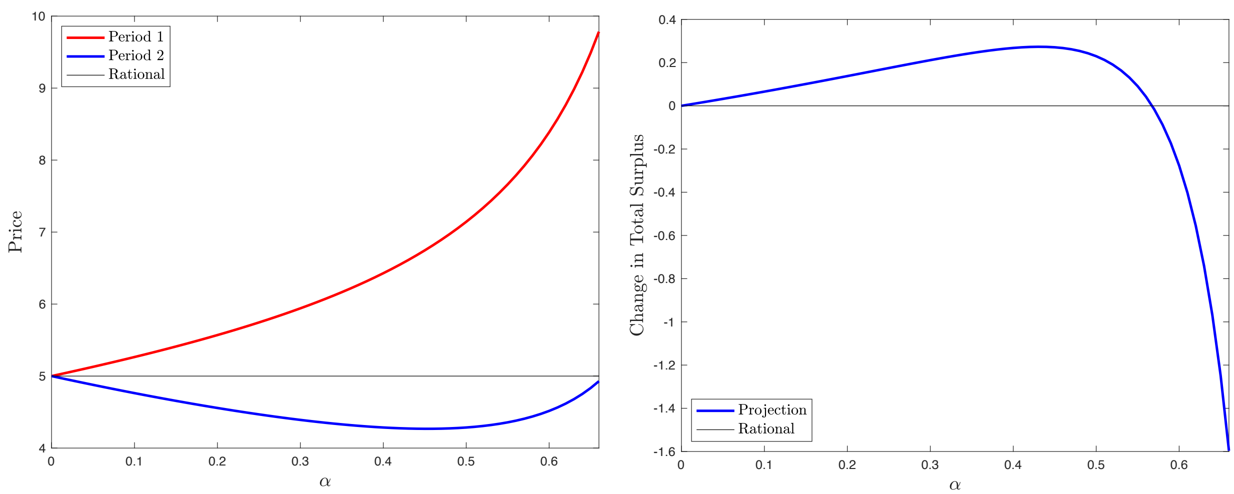
To further elucidate the pricing and welfare effects of projection, suppose T is uniform on $[\underline{t}, \bar{t}]$. In this case, the demands of informed and uninformed agents are

$$D^I\big(p;\bar{\omega}(s)\big) = \frac{\bar{\omega}(s) + \bar{t} - p}{\bar{t} - \underline{t}} \quad \text{and} \quad D^U(p;\bar{\omega}_2) = \frac{\bar{\omega}_2 + (1-\alpha)\bar{t} - p}{(1-\alpha)(\bar{t} - \underline{t})}, \tag{16}$$

respectively, where $\bar{\omega}_2 = (1-\alpha)\bar{\omega}(s) + \alpha p_1$. It is straightforward to show that the interior solution is such that $p_1^* > p^M > p_2^*$. Panel (a) of Figure 4 shows how each p_n^* changes with α.[33] Intuitively,

[33]In this example, $\bar{t} = 10$, $\underline{t} = -10$, $\bar{\omega}(s) = 0$. We plot outcomes for $\alpha \leq 2/3$ since this is the region that admits an

$p_n^* \to p^M$ for both $n = 1, 2$ as $\alpha \to 0$. As α increases, p_1 has a stronger positive effect on the beliefs of Generation 2, and hence p_1^* increases in α. By contrast, p_2^* is not monotone in α. Since the consumers who would be submarginal at p_1^* are those with inflated beliefs, p_2^* will necessarily fall below p_1^*. Moreover, when α is small, the perceived valuations of consumers in Generation 2 exhibit near-rational levels of variation, so a reduction in p_2 will not attract many more buyers than it would under rational learning. Hence, there is little benefit in deviating from the rational monopoly price. But as α increases, perceived valuations become more clustered around $\bar{\omega}_2$, meaning that a price drop will attract a bigger proportion of the market and will thus be more profitable. This explains why p_2^* initially decreases in α. However, once α is sufficiently large—and thus beliefs are substantially inflated due to a high p_1^*—the seller can capture a significant fraction of the market with a smaller deviation from p^M.



(a) Prices (p_n)

(b) Change in Total Surplus

Figure 4: *Optimal prices and the effect of projection on total surplus as a function of* α

While harming the surplus of uninformed consumers, projection can actually increase total surplus so long as the bias is not too severe. Indeed, total quantity demanded across both periods can be higher under projection than the rational benchmark. This reduces the traditional deadweight loss due to monopoly pricing. However, this inflated level of sales can sometimes be detrimental to total surplus, since sufficiently strong projection can induce inefficient over-consumption (as in Proposition 6). Panel (b) of Figure 4 shows how total surplus changes as a function of α; total surplus begins to fall once sales have expanded to the point that those with negative valuations are lured into buying.

interior solution (shown in the figure). For $\alpha > 2/3$, we necessarily have a corner solution at which the seller sets p_1 at the price ceiling (see footnote 29).

4.2.2 Arbitrary Horizon

We now demonstrate how our declining-price result extends beyond $N = 2$. Namely, we show that the initial price is inflated above the static monopoly price and that prices gradually decline thereafter. This result follows from a novel trade-off the seller faces in any given period (aside from the first or last). On the one hand, keeping the price high and restraining current sales helps to maintain inflated beliefs further into the future. On the other hand, lowering the current price allows the seller to reap high current sales by exploiting the inflated beliefs generated by high prices in previous periods. This inter-temporal trade-off results in a declining optimal price path.

For this analysis, we continue to focus on the case in which private values are uniformly distributed over $[\underline{t}, \overline{t}]$, and we restrict attention to interior cases where it is never optimal to serve the lowest type (which amounts to assuming $\underline{t}$ is sufficiently low).[34] Equation (14) implies that the aggregate biased beliefs evolve according to

$$\bar{\omega}_{n+1} = \lambda\big[(1-\alpha)\bar{\omega}(s) + \alpha p_n\big] + (1-\lambda)\bar{\omega}_n. \tag{17}$$

We know from Proposition 3 that the aggregate biased belief in Generation n is increasing in all past prices; the following lemma provides an explicit expression of this aggregate belief for the case of uniformly distributed tastes.

Lemma 2. *Suppose (p_k, s) admits interior demand for all $k \leq n$. The aggregate belief in period n is $\bar{\omega}_n = (1-\alpha)\bar{\omega}(s) + \alpha\tilde{p}^{n-1}$, where $\tilde{p}^{n-1}$ is a weighted average of past prices:*

$$\tilde{p}^{n-1} \equiv (1-\lambda)^{n-2}p_1 + \sum_{k=2}^{n-1} \lambda(1-\lambda)^{n-1-k}p_k. \tag{18}$$

Since the weights on all past prices in (18) sum to one (by virtue of being a weighted average), the overall effect of past prices on $\bar{\omega}_n$ is always equal to α. Notably, however, more recent prices have a bigger impact on the current belief than earlier ones.

The "stock variable" $\tilde{p}^{n-1}$ captures the sway of past prices on current beliefs. As such, it is convenient to re-write the demand of Generation n in terms of $\tilde{p}^{n-1}$ rather than $\bar{\omega}_n$. From (13) and Lemma 2, demand in period n as a function of each previous price is

$$D(p_n; \tilde{p}^{n-1}, \bar{\omega}(s)) = \frac{(1-\alpha)\left(\overline{t} + \bar{\omega}(s)\right) + \alpha(1-\lambda)\tilde{p}^{n-1} - (1-\lambda\alpha)p_n}{(1-\alpha)(\overline{t} - \underline{t})}. \tag{19}$$

Given the objective function in (15), we then arrive at the following first-order condition for the

[34] With uniform tastes, our usual assumption that (p^M, s) admits interior demand is equivalent to $\bar{\omega}(s) + \overline{t} > 0$ and $\bar{\omega}(s) < \overline{t} - 2\underline{t}$. It is never optimal to serve the lowest projecting type if we also have $(1-\alpha)\bar{\omega}(s) + \alpha\bar{p} < \overline{t} - 2\underline{t}$.

price in a non-terminal period $n \geq 2$:

$$p_n = \frac{1}{1 - \lambda\alpha}\left((1-\alpha)p^M + \frac{\alpha(1-\lambda)}{2}\left[\tilde{p}^{n-1} + \sum_{k=n+1}^{N} p_k \frac{\partial \tilde{p}^{k-1}}{\partial p_n}\right]\right), \tag{20}$$

where we've used the fact that $p^M = \left(\bar{t} + \bar{\omega}(s)\right)/2$ when (p^M, s) admits interior demand. The term in squared brackets in Equation (20) highlights the intertemporal incentives in pricing. Namely, the seller has a greater incentive to inflate the current price in order to manipulate future consumers' beliefs when the current period is earlier in the horizon, and thus influences a greater number of subsequent generations. This leads to an optimal price path that declines over time.

Proposition 7. *Consider any $\alpha > 0$ and any s such that (p^M, s) admits interior demand.*

1. *The initial price is inflated: $p_1^* > p^M$.*
2. *The optimal price path is declining: For all $n \geq 2$, we have $p_n^* < p_{n-1}^*$.*

As discussed above, this result follows from the seller balancing the trade-off between manipulating the beliefs of future consumers by maintaining a high current price versus exploiting consumers' current beliefs by undercutting the previous price. While our model introduces a clear incentive to initially inflate the price and then drop it, it does not predict occasional sales where the price temporally drops and then returns to a high level. Rather, we predict a gradual decline in prices, which is consistent with the pricing pattern observed for novel products (e.g., a new smartphone or a new fitness program), where consumers are uncertain about the product's quality; see Bayus (1992), Krishnan et al. (1999), Jain et al. (1999), Nair (2007), and Liu (2010).

Figure 5 provides an example of the optimal price path for $N = 20$ for different degrees of projection. Intuitively, the extent to which prices deviate from the monopoly price increases when α is high, since in this case prices have more sway on beliefs. Although it's not captured in Figure 5, a similar intuition holds as λ decreases: deviating from the monopoly price is less costly when there are fewer informed agents.

This declining price path has natural implications for the evolution of aggregate beliefs and demand, as well. Since the current aggregate belief is a convex combination of the previous belief and price, a declining price path implies that beliefs also decline over time: later generations of consumers perceive a lower quality, on average, than earlier generations. Additionally, the quantity demanded in periods with distorted beliefs (i.e., for period 2 onward) is "U-shaped": the inflated price in the first period leads Generation 2 to demand an aggregate quantity above the rational benchmark. However, as the price levels off near the rational monopoly price, the aggregate demand converges to the rational monopoly level.[35] Finally, near the end of the horizon—once there is

[35]This reflects the fact that aggregate demand in the steady state of our model matches the aggregate demand under

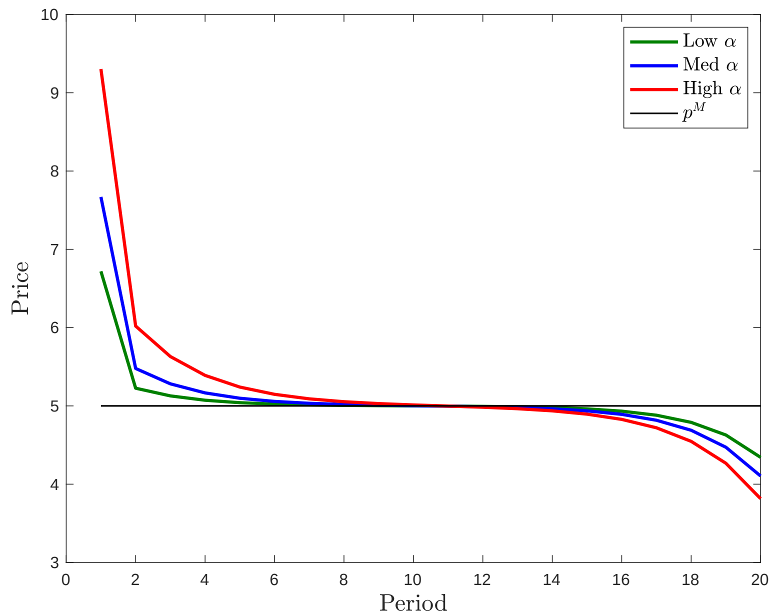


Figure 5: Example Price path for $N = 20$ for various degrees of projection. The example assumes $\bar{t} = 10$ and $\bar{\omega}(s) = 0$.

little remaining incentive to maintain high prices to manipulate future generations—the seller will lower the price below p^M since the market demand has become more elastic, which again leads to significantly more sales than the rational monopoly benchmark.

5 Extensions and Further Applications

In this section, we discuss further implications of taste projection when we relax our assumptions that consumers (i) are short lived and (ii) have unit demand.

5.1 Endogenous Timing and Underappreciation of Selection Effects

Section 4 showed how high-to-low pricing can induce "short-lived" low-valuation projectors to excessively adopt the good. We now show that such over-adoption can arise even if the price is fixed and "long-lived" consumers can time their purchase. Thus, the idea that projection causes consumers to be overly influenced by earlier purchases is not limited to settings with changing prices.

To demonstrate the logic, we consider a two-period model. Uninformed consumers with low private values defer their decisions until the second period in order learn from the quantity demanded by early-adopters. But since they fail to appreciate the difference in tastes between themselves and

rational learning (Section 3). Hence, when the price is near constant for many periods, the resulting quantity demanded converges to the rational level given that (near) constant price; see Proposition 4.

those with an incentive to adopt early, they treat high initial demand as an overly-optimistic signal about the good's quality. As such, they systematically over-consume and face greater disappointment relative to the rational benchmark. This application therefore speaks to the empirical finding that late adopters exhibit greater disappointment with a product, as reflected by declining consumer reviews (e.g., Li and Hitt, 2008; Dai et al., 2018). We argue that taste projection provides a specific mechanism for why selection effects may be under-appreciated in this particular context: while projectors understand that there is selection across periods, they systematically underestimate the strength of this effect.

More formally, consider a two-period variant of our dynamic model from Section 4. Instead of a new mass of consumers entering each period, there is a single group of consumers with unit demand who can buy in either period (or not at all). We focus on the case where the price p is fixed across periods. We additionally assume T is uniform to ease exposition, but the logic will transparently generalize. Finally, as above, a fraction λ of consumers observe s while $1-\lambda$ are uninformed.

Informed agents buy in period 1 or never, since they have nothing to learn from delaying; they buy immediately if $\bar{\omega}(s)+t \geq p$.[36] Uninformed agents with low private values may defer their purchase decision to period 2 in order to learn from those adopting in period 1. Specifically, an uninformed agent buys in period 1 if $\bar{\omega}_0 + t \geq p$, where $\bar{\omega}_0$ reflects the expected quality among uninformed agents.[37] Otherwise, they observe the quantity demanded in period 1, form an updated expectation $\hat{\omega}$, and then buy in period 2 if $\hat{\omega}+t \geq p$.

The quantity demanded in period 1 is $d_1 = \lambda D(p;\bar{\omega}(s)) + (1-\lambda)D(p;\bar{\omega}_0)$ where $D(p;\omega) = 1-F(p-\omega)$. As usual, a projecting agent in period 2 with taste t updates their belief to $\hat{\omega}_2(t)$, which is the value $\hat{\omega}$ that fits their model to the observed outcome: $\hat{\omega}$ solves $d_1 = \lambda\widehat{D}(p;\hat{\omega}|t) + (1-\lambda)\widehat{D}(p;\bar{\omega}_0|t)$, where $\widehat{D}(p;\omega|t) = 1-\widehat{F}(p-\omega|t)$. To state our result, we impose some convenient technical assumptions to ensure that there are well-defined marginal types in period 2 under both rational inference and projection, denoted by t_2^* and $\hat{t}_2$, respectively. Namely, suppose that $D(p;\bar{\omega}_0) \in (0,1)$, $\widehat{D}(p;\bar{\omega}_0|\underline{t}) > 0$, and $d_1 \leq \lambda + (1-\lambda)\widehat{D}(p;\bar{\omega}_0|\underline{t})$. The first condition means that an interior fraction of uninformed agents delay. The final two conditions mean that all projectors expect an interior fraction to delay and the observed demand is consistent with their models; this happens when λ is sufficiently large compared to α.

Proposition 8. *Suppose (p,s) admits interior demand and $\lambda > \alpha > 0$.*

1. *Suppose informed agents have positive information about the good; i.e., $\bar{\omega}(s) > \bar{\omega}_0$. (i) The quantity demanded in period 2 exceeds the rational benchmark, and the range of types who*

[36] This relies on a mild (unmodeled) assumption that delaying consumption is costly to consumers, or that indifference is broken in favor of buying sooner rather than later.

[37] Our conclusions in this application would not change if $\bar{\omega}_0$ were to depend on p—which might naturally occur if p partially signals quality—so long as informed consumers have additional information that is not revealed by p.

suboptimally adopt, $\left[\hat{t}_2, t_2^*\right]$, is increasing in both α and $\bar{\omega}(s)-\bar{\omega}_0$. (ii) There exists a threshold value $\tilde{t} > t_2^*$ such that all types $t \in \left[\hat{t}_2, \tilde{t}\right]$ will, on average, receive lower quality than they expect; i.e., $t < \tilde{t}$ implies $\mathbb{E}[\omega - \hat{\omega}_2(t)|s] < 0$.

2. *Suppose informed agents have negative information about the good; i.e., $\bar{\omega}(s) < \bar{\omega}_0$. Then there is zero demand in period 2, as in the rational benchmark.*

Proposition 8 stems from projectors underestimating a selection effect that naturally emerges in this environment: consumers who decide to buy in period 1 tend to have higher private values than those who delay. Those who delay are aware of this selection effect, but they underestimate it. Since the delayers systematically underestimate the private values of those with stronger tastes than them, they over-attribute observations from period 1 to quality rather than differences in tastes. When d_1 is stronger than expected, delayers become too optimistic and too many of them buy—they are subsequently disappointed by the quality they receive. When d_1 is weaker than expected, delayers become too pessimistic and don't buy. However, they would not buy based on this bad news even under rational learning: since they were unwilling to buy with belief $\bar{\omega}_0$, they are only willing to buy in period 2 if they receive good news. Hence, projection generates an asymmetric bias in behavior, leading to over-adoption among delayers, but not under-adoption. Additionally, insofar as unmet quality expectations drive negative product reviews, the fact that over-adoption is coupled with systematic disappointment suggests that high initial reviews for a product will too frequently be followed by negative reviews (Li and Hitt, 2008; Papanastasiou et al., 2015; Dai et al., 2018).

5.2 Multi-Unit Demand

We now revisit the static equilibrium from Section 3 but allow consumers to have multi-unit demand. As before, consumers still form type-dependent beliefs that are negatively related to their tastes. In contrast to that previous case, however, projectors now fine-tune their actions to their erroneous beliefs. Thus, all projecting types will generically consume a sub-optimal amount in equilibrium, leading to potentially large inefficiencies. In particular, since perceptions are negatively related to tastes, high types underconsume while low types overconsume.[38]

For simplicity, we consider the familiar case of quadratic utility (e.g., Judd and Riordan, 1994; Caminal and Vives, 1996), where a consumer's valuation for x units of the good is given by $u(x;\omega,t) = (\omega+t)x - x^2/2$. A consumer with a quality expectation of $\hat{\omega}$ facing a per-unit price of p then demands a quantity $x^*(p;\hat{\omega},t) = \hat{\omega} + t - p$ if $\hat{\omega} + t - p \geq 0$ and $x^*(p;\hat{\omega},t) = 0$ otherwise.

[38] Although this is straightforward given our earlier results on biased perceptions, it is nevertheless important to verify whether projection indeed creates steady-state inefficiencies. The reason such inefficiencies were absent in Section 3 was merely an artifact of the unit-demand structure.

As in Section 3, a fraction λ of consumers observe s and form a quality expectation of $\bar{\omega}(s)$. The remaining fraction $1-\lambda$ form this expectation based on the aggregate demand at price p. The steady-state equilibrium is analogous to the one defined above: uninformed agents make inferences that are consistent with the observed quantity demanded and their misspecified model, and the resulting quantity is consistent with those beliefs. More specifically, let $\hat{\omega}(t)$ be type t's quality expectation in equilibrium; this type will then demand $x^*\big(p; \hat{\omega}(t), t\big)$ units. The aggregate demand in equilibrium is thus

$$d = \lambda \cdot \underbrace{\int_{\mathcal{T}} x^*\big(p; \bar{\omega}(s), t\big) dF(t)}_{\text{Informed Demand}} + (1-\lambda) \cdot \underbrace{\int_{\mathcal{T}} x^*\big(p; \hat{\omega}(t), t\big) dF(t)}_{\text{Uninformed Demand}}. \tag{21}$$

Since uninformed agents expect that all types reach a common and correct expectation of ω in equilibrium, each $\hat{\omega}(t)$ is the value that predicts quantity d under type t's model given the presumption that all types have inferred this same value.

Proposition 9. *Suppose (p, s) admits positive aggregate demand among informed consumers. For any $\alpha > 0$, there exists a unique equilibrium profile of beliefs, $\hat{\omega}(t)$, which has the following properties:*

1. *Quality perceptions are negatively related to tastes: $\hat{\omega}(t)$ is strictly decreasing in t.*
2. *Relative to the rational benchmark, high types demand too little and low types demand too much: there exists an interior threshold type $\tilde{t}$ such that $t > \tilde{t}$ implies that $x^*\big(p; \hat{\omega}(t), t\big) < x^*\big(p; \bar{\omega}(s), t\big)$ and $t < \tilde{t}$ implies that $x^*\big(p; \hat{\omega}(t), t\big) > x^*\big(p; \bar{\omega}(s), t\big)$.*
3. *Relative to the rational benchmark, demand along the extensive margin increases: the lowest uninformed type who buys a positive quantity is lower than the lowest type who buys a positive quantity in the rational benchmark.*
4. *More extreme types exhibit greater inefficiency: $\big|x^*\big(p; \hat{\omega}(t), t\big) - x^*\big(p; \bar{\omega}(s), t\big)\big|$ is strictly increasing in $|t - \tilde{t}|$.*

The intuition for Part 1 of Proposition 9 is identical to the unit-demand case. However, consumers now tailor their individual demand to their idiosyncratic beliefs. This underlies Part 2: since high types are typically pessimistic about the product's quality, they consume too little; low types instead consume too much. In this sense, consumption along the intensive margin is reduced, since projection reduces the quantity demanded among the high types who consume the most. But consumption along the *extensive* margin increases (Part 3). That is, the set of types who consume the good in equilibrium expands: some low types who would entirely abstain under rational inference are now persuaded to buy the product. Parts 2 and 3 together imply that, relative to the rational benchmark, consumption is spread more thinly across a wider range of buyers.

The logic behind these results is quite transparent as $\alpha \to 1$. In this case, observers think there is essentially no heterogeneity in tastes, and that aggregate demand derives from all individuals consuming roughly the same quantity. From a projector's point of view, the average quantity demanded is then a near perfect signal about how much he himself should consume—he should consume that same amount, since he is just like everybody else. Thus, in equilibrium, the difference in consumption across types narrows, while the set of types who consume expands.

Finally, among the segment of consumers who buy in equilibrium, those with types closer to the extremes make worse decisions (Part 4). Intuitively, these types are farther from the average buyer, and thus their mental model provides a worse interpretation of the data. A truly average projecting consumer is fairly accurate when she imagines that most people share her tastes. But those with more extreme tastes form a more distorted model of the world when assuming their tastes are typical. Proposition 9, along with the results of Section 4, reveal that where the burden of projection falls depends on the demand structure: with single-unit demand, it is only low types who can be manipulated into inefficiently adopting a product; with multi-unit demand, the burden falls on extreme types, either high or low.

6 Conclusion

Evidence suggests that people often misperceive others' tastes, attitudes, and motives by exaggerating the similarity between others and themselves. In this paper, we have examined some implications that arise when consumers interpret market data through the lens of these misperceptions. In contexts where consumers aim to learn the commonly-valued quality of a product from others' demand, we showed that projection leads to systematically distorted beliefs. Namely, projecting consumers will form estimates of the quality that are negatively related to their tastes, and these estimates are increasing in the product's price. These misinferences create new pricing incentives for a monopolistic seller: in a dynamic setting, the seller will charge high initial prices to inflate future consumers' beliefs and then will gradually lower the price to capitalize on these distorted beliefs. Projection also has implications for efficiency. For instance, either the seller's manipulative pricing or a failure to appreciate selection effects can lead projectors to over-adopt a good even when such adoption is inefficient under rational learning.[39]

There are several other potential applications of our framework. As discussed above, projection leads to lower dispersion in consumers' valuations and hence to a counter-clockwise rotation of the market demand curve. Johnson and Myatt (2006) study how demand rotations influence various features of a monopolist's marketing strategies. In this sense, the insights from Johnson and Myatt

[39]It is worth emphasizing that our statements about efficiency implicitly disregard externatlities; indeed, projection could be beneficial from a social-welfare perspective when large-scale adoption is a critical objective (e.g., adoption of clean-energy technologies). We leave this analysis for future work.

(2006) should apply to a market with projectors. For instance, in a setting where the seller engages in second-degree price discrimination by offering a menu of multi-unit bundles, they show that a counter-clockwise rotation of the demand curve can lead the seller to prefer a smaller menu. Thus, a seller should have a similar preference when facing projecting consumers.

Finally, projection may also distort an individual's perception of her information sources in various ways. For instance, consider an individual who is uncertain about the variance in signals conditional on ω and updates her belief over this value after consuming the good and learning ω. This belief revision will depend on the deviation between ω and her expectation, $\hat{\omega}(t)$. Since $\hat{\omega}(t)$ is typically biased, projectors will, on average, perceive greater deviations between the realized quality and their expectations, leading them to overestimate the variance in signals. Thus, projectors may come to underweight valuable information. Alternatively, suppose consumers entertain the possibility that others may be biased in favor of a particular option (e.g., a particular brand, author, or politician), supporting it even when they know it has low quality. If a projector forms beliefs about whether such a bias exists ex post, she will be predisposed to think others are systematically biased against options that suit her tastes. This is because the observed popularity of the option will be inconsistent with a projector's misspecified model once she learns its true quality. For example, a projector who realizes that she dislikes an option will observe a stronger demand than expected; she may therefore conclude that others' support stems from some ulterior motive, neglecting that it may come from mere differences in tastes. Such skepticism of others' motives may shed light on why some groups are unmoved by others' actions even when they reveal valuable information.

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Appendix

A Alternative Signal Structures

In this appendix, we show that our key comparative statics emerge in settings with richer heterogeneity in private information. We also note a few additional implications that emerge in these settings.

A.1 Fully-Heterogeneous Private Signals

In this section, we consider the case in which each agent receives a private signal correlated with ω. We show that a projector's inferred quality upon observing the aggregate quantity demanded by these privately informed agents is still: (i) negatively related to her taste; and (ii) positively related to the price that predecessors paid. We will show this in a two-period model similar to Section 4.

As in the main text, suppose that individuals share a common prior over ω with support $\mathbb{R}$. In each generation $n = 1, 2$, individual i observes the realization of a private signal $S_{i,n}$ that is correlated with ω. We assume that signals are i.i.d. across all individuals in both periods, and that no signal realization perfectly reveals ω. Let $Z_{i,n} \equiv \mathbb{E}[\omega|S_{i,n}]$ denote a consumer's "private belief"—their expected quality conditional on their signal and the prior. We work directly with the distribution of $Z_{i,n}$ conditional on ω rather than conditional distributions over signals. As such, let $Z(\omega)$ denote the random variable representing individuals' private beliefs conditional on ω. We assume that $Z(\omega)$ can be expressed as $Z(\omega) = m(\omega) + Y$ for some strictly increasing function m and a random variable Y that is independent of ω (and T) and has a log-concave density.[40] This implies that consumers' interim valuations for the good in period 1 are distributed according to $V(\omega) \equiv m(\omega) + Y + T$. Let $H(\cdot;\omega)$ denote the CDF of $V(\omega)$. In period 1, individuals act on their private signals alone. Thus, the demand function in period 1 is $D_1(p;\omega) \equiv 1 - H(p_1;\omega)$.

Fixing the true quality ω, we are interested in the quality inferred by consumers in period 2 upon observing $d_1 = D(p_1;\omega)$ and price p_1. Let $\hat{\omega}(t;p_1)$ denote the quality inferred by a consumer with taste t.

Proposition A.1 (Comparative Statics in the Heterogeneous-Signal Model)**.** *Consider the signal structure of Section A.1. Fix ω, and consider any p_1 such that demand in period 1 is interior (i.e., $d_1 \in (0,1)$). For any $\alpha > 0$, the inferred quality of a projector with type t who observes d_1 is: (i) decreasing in t (ii) increasing in p_1.*

The proof, presented below, follows a similar logic to the graphical argument in Figure 1. Since a projector thinks interim valuations are less dispersed than they truly are, her perceived demand curve intersects the true demand curve at a point where the perceived demand curve has a greater price elasticity. Thus, to explain a market outcome at a higher price, the projector must consider a demand curve that is shifted outward relative to the initial perceived demand. This outward shift corresponds to a higher perceived quality. The key difference between this case and the one considered in the main text is that the observed quantity demanded now results from both variation in consumers' tastes and variation in their signals. We therefore make use of results on the "dispersion ordering" of convolutions of log-concave random variables to prove that, even when consumers'

[40] This structure nests the familiar Gaussian structure noted in the main text, but is also more general.

have disperse private information, the perceived and true demand curves continue to obey a single-crossing property crucial to the logic depicted in Figure 1.

Proof of Proposition A.1. Fix ω, and consider any p_1 such that the quantity demanded in period 1 is interior (i.e., $d_1 \in (0,1)$). We examine how $\hat{\omega}(t;p_1)$ varies in t and p_1. Note that $\hat{\omega}(t;p_1)$ is the value of $\hat{\omega}$ that solves $\widehat{D}_1(p_1;\hat{\omega}|t) = D_1(p_1;\omega)$, where $\widehat{D}_1(p_1;\hat{\omega}|t)$ is type t's misperceived demand function: $\widehat{D}_1(p_1;\hat{\omega}|t) = 1 - \widehat{H}(p;\hat{\omega}|t)$, and $\widehat{H}(\cdot;\hat{\omega}|t)$ is the CDF of $\widehat{V}(\hat{\omega}|t) \equiv m(\hat{\omega}) + Y + \widehat{T}(t)$. Hence $\hat{\omega}(t;p_1)$ is the value of $\hat{\omega}$ that solves $L(\hat{\omega};t,p_1) \equiv \widehat{D}_1(p_1;\hat{\omega}|t) - D_1(p_1;\omega) = 0$.

Part 1: The Effect of t on Perceived Quality. By the Implicit Function Theorem (IFT):

$$\frac{\partial\hat{\omega}(t;p_1)}{\partial t} = -\frac{\partial L(\hat{\omega};t,p_1)}{\partial t}\left(\frac{\partial L(\hat{\omega};t,p_1)}{\partial\hat{\omega}}\right)^{-1}\Bigg|_{\hat{\omega}=\hat{\omega}(t;p_1)}. \tag{A.1}$$

Notice that, for any p_1 that generates interior demand and any t, $\frac{\partial}{\partial\hat{\omega}}L(\hat{\omega};t,p_1) = \frac{\partial}{\partial\hat{\omega}}\widehat{D}_1(p_1;\hat{\omega}|t) > 0$ given our mild assumption that demand is increasing in quality (i.e., m is a strictly increasing function). Thus

$$\operatorname{sgn}\left(\frac{\partial\hat{\omega}(t;p_1)}{\partial t}\right) = \operatorname{sgn}\left(-\frac{\partial L(\hat{\omega};t,p_1)}{\partial t}\bigg|_{\hat{\omega}=\hat{\omega}(t;p_1)}\right). \tag{A.2}$$

Note that

$$-\frac{\partial L(\hat{\omega};t,p_1)}{\partial t} = -\frac{\partial}{\partial t}\widehat{D}_1(p_1;\hat{\omega}|t) < 0. \tag{A.3}$$

This follows from the fact that $t' > t$ implies that $\widehat{V}(\hat{\omega}|t')$ first-order stochastically dominates $\widehat{V}(\hat{\omega}|t)$ since in this case $\widehat{T}(t')$ first-order stochastically dominates $\widehat{T}(t)$; accordingly, $\widehat{H}(p;\hat{\omega}|t)$ is decreasing in t and thus $\widehat{D}_1(p;\hat{\omega}|t)$ is increasing in t.

Part 2: The Effect of p on Perceived Quality. Invoking the IFT again, the discussion following (A.1) implies that

$$\operatorname{sgn}\left(\frac{\partial\hat{\omega}(t;p)}{\partial p}\right) = \operatorname{sgn}\left(-\frac{\partial L(\hat{\omega};p)}{\partial p}\bigg|_{\hat{\omega}=\hat{\omega}(t;p_1)}\right). \tag{A.4}$$

Note that

$$-\frac{\partial L(\hat{\omega};p)}{\partial p} = \frac{\partial}{\partial p}D_1(p;\omega) - \frac{\partial}{\partial p}\widehat{D}_1(p;\hat{\omega}|t). \tag{A.5}$$

With downward-sloping demand functions, the previous expression is positive when evaluated at $\hat{\omega}(t;p_1)$ if and only if

$$\left|\frac{\partial}{\partial p}D_1(p_1;\omega)\right| < \left|\frac{\partial}{\partial p}\widehat{D}_1(p_1;\hat{\omega}(t;p_1)|t)\right|; \tag{A.6}$$

that is, if and only if the perceived demand function is locally more price sensitive at the original market outcome than the true demand function.

Since $\hat{\omega}(t;p_1)$ is a state in which type t's perceived demand curve intersects the true demand curve at the observed market outcome (d_1,p_1) (i.e., $\widehat{D}_1(p_1;\hat{\omega}(t;p)|t) = d_1 = D_1(p_1;\omega)$), a sufficient condition for Condition (A.6) is that for any arbitrary $\hat{\omega}$, $\widehat{D}_1(\cdot;\hat{\omega}|t)$ crosses $D_1(\cdot;\omega)$ at most once and does so from above. That is, there exists at most one price p^* such that $\widehat{D}_1(p^*;\hat{\omega}|t) = D_1(p^*;\omega)$, and p^* is such that $\widehat{D}_1(p;\hat{\omega}|t) < D_1(p;\omega)$ for all $p > p^*$ and $\widehat{D}_1(p;\hat{\omega}|t) > D_1(p;\omega)$ for all $p < p^*$. (Note that the demand curves in Figure 1 are drawn, as usual, with p on the y-axis; from that perspective, the previous condition implies that the perceived demand curve crosses the true one from below.)

To complete the proof, we prove the sufficient condition above: for any arbitrary $\hat{\omega}$ and t, there exists at most one price p^* such that $\widehat{D}_1(p^*;\hat{\omega}|t) = D_1(p^*;\omega)$, and p^* is such that $\widehat{D}_1(p;\hat{\omega}|t) < D_1(p;\omega)$ for all $p > p^*$ and $\widehat{D}_1(p;\hat{\omega}|t) > D_1(p;\omega)$ for all $p < p^*$. Given that $D_1(p;\omega) = 1 - H(p;\hat{\omega})$ and $\widehat{D}_1(p;\hat{\omega}|t) = 1 - \widehat{H}(p;\hat{\omega}|t)$, it suffices to show that $\widehat{H}(p|\hat{\omega};t)$ crosses $H(p|\omega)$ at most once and does so from below (i.e., there exists at most one price p^* such that $\widehat{H}(p|\hat{\omega};t) < H(p;\omega)$ if $p < p^*$ and $\widehat{H}(p|\hat{\omega};t) > H(p;\omega)$ if $p > p^*$).

We prove this using the concept of dispersive order defined by Shaked (1982) and Shaked and Shanthikumar (2007). For any two arbitrary random variables X and Y with CDFs F_X and F_Y, we say that X is less dispersed than Y, denoted $X \leq_{disp} Y$, if $F_X^{-1}(b) - F_X^{-1}(a) \leq F_Y^{-1}(b) - F^{-1}(a)$ whenever $0 \leq a \leq b \leq 1$. By Theorem 2.1 of Shaked (1982), $X \leq_{disp} Y$ iff F_X crosses F_Y at most once and does so from below. Thus, it suffices to show that $\widehat{V}(\hat{\omega};t) \leq_{disp} V(\omega)$, which is equivalent to $\widehat{T}(t) + Z(\hat{\omega}) \leq_{disp} T + Z(\omega)$. Since $Z(\omega) = m(\omega) + Y$, the previous condition is equivalent to $\widehat{T}(t) + m(\hat{\omega}) + Y \leq_{disp} T + m(\omega) + Y$, where $m(\hat{\omega})$ and $m(\omega)$ are constants given that we are conditioning on ω and $\hat{\omega}$. As noted in Comment 3.B.2 of Shaked and Shanthikumar (2007), the order $\leq_{disp}$ is location invariant, meaning that $\widehat{T}(t) + m(\hat{\omega}) + Y \leq_{disp} T + m(\omega) + Y \Leftrightarrow \widehat{T}(t) + Y \leq_{disp} T + Y$. Since Y has a log-concave density and is independent of T and $\widehat{T}(t)$, Theorem 3.B.8 of Shaked and Shanthikumar (2007) implies that $\widehat{T}(t) + Y \leq_{disp} T + Y$ if $\widehat{T}(t) \leq_{disp} T$. Thus, to complete the proof it suffices to show that $\widehat{T}(t) \leq_{disp} T$. Again by Theorem 2.1 in Shaked (1982), this holds so long as $\widehat{F}(\cdot|t)$ crosses F only once and does so from below. This is true by Part 4 of Observation 1, completing the proof. ∎

A.2 Heterogeneous Signals Across Periods

In this section, we consider a structure in which each generation of consumers observes a distinct signal. All consumers in each Generation n observe the same signal realization, which we denote by s_n. We assume that s_n is i.i.d. for all n. Furthermore, s_n is "quasi-public": it is observed by all agents within Generation n, but not by agents in any other generation.[41] As in the main text (and the previous appendix section), we again show that the perceived quality of each agent in each Generation $n \geq 2$ is: (i) negatively related to their taste; and (ii) positively related to the price that predecessors paid.

Setup. Agents in Generation n attempt to infer the posterior beliefs of agents in period $n-1$ from their quantity demanded. If agents are rational, then all agents in each generation hold a common expectation over ω. Let $\tilde{\omega}_{n-1}$ denote this rational expectation among Generation $n-1$ for $n \geq 2$. Agents in Generation n can then perfectly extract $\tilde{\omega}_{n-1}$ from the observed market coverage in Generation $n-1$ (assuming this value is interior).

To make matters concrete, we consider the familiar Gaussian information structure: $\omega \sim N(\bar{\omega}_0, \rho^2)$, and $s_n \sim N(\omega, \eta^2)$. Rational updating then takes the form

$$\tilde{\omega}_n = \gamma_n s_n + (1-\gamma_n)\tilde{\omega}_{n-1}, \quad \text{where} \quad \gamma_n = \frac{1}{n + \eta^2/\rho^2}. \tag{A.7}$$

[41] If generations consisted of a single agent, this structure would resemble the canonical sequential herding model (e.g., Banerjee, 1992; Bikhchandani et al., 1992; Smith and Sørensen, 2000).

As the updating process in A.7 suggests, a rational Generation n will combine their own signal, s_n, with the inferred posterior belief of Generation $n-1$, $\tilde{\omega}_{n-1}$, to reach their posterior estimate of ω.

With projection, an agent in Generation n *thinks* he can perfectly extract the posterior expectation of ω held by the previous generation, but does so incorrectly. As usual, his incorrect inference will depend on his taste, t. Denote this (mis)extracted value of $\tilde{\omega}_{n-1}$ by $\hat{\omega}_{n-1}(t)$. The projector will then use A.7 to form a posterior estimate of $\gamma_n s_n + (1-\gamma_n)\hat{\omega}_{n-1}(t)$. Below, we analyze how projectors' beliefs evolve within this structure.

We first consider how beliefs evolve within the first few periods. For simplicity, we normalize $\bar{\omega}_0 = 0$. Since Generation 1 does not observe others, their is no scope for mislearning in period 1. Hence, agents in Generation 1 share a common (rational) estimate of ω equal to $\tilde{\omega}_1 = \gamma_1 s_1$. Thus, an agent buys iff $\tilde{\omega}_1 + t_i \geq p_1 \Leftrightarrow t_i \geq p_1 - \tilde{\omega}_1$, and hence demand in period 1 is

$$D_1(p_1; \tilde{\omega}_1) = 1 - F(p_1 - \tilde{\omega}_1). \tag{A.8}$$

Distorted Beliefs in Generation 2. An agent in Generation 2 with taste t thinks that, conditional on Generation 1 holding a posterior expectation of $\hat{\omega}$, their demand is given by

$$\widehat{D}_1(p_1; \hat{\omega}|t) = 1 - \widehat{F}(p_1 - \hat{\omega}|t) = 1 - F\left(\frac{p_1 - \hat{\omega} - \alpha t}{1-\alpha}\right). \tag{A.9}$$

This agent wrongly infers that the posterior expectation in Generation 1 is the value of $\hat{\omega}$ that solves $D(p_1; \tilde{\omega}_1) = \widehat{D}(p_1, \hat{\omega}|t)$, which we denote by $\hat{\omega}_1(t)$. Hence,

$$\hat{\omega}_1(t) = (1-\alpha)\tilde{\omega}_1 + \alpha(p_1 - t). \tag{A.10}$$

This misperception is identical to the one formed by agents in Generation 2 of the baseline model in the main text (see Equation 10). Furthermore, given that $\tilde{\omega}_1 = \gamma_1 s_1$, the preceding equation implies that an agent with taste t misinfers the signal to be

$$\hat{s}_1(t) = (1-\alpha)s_1 + \frac{1}{\gamma_1}\alpha(p_1 - t). \tag{A.11}$$

An immediate implication of (A.10) and (A.11) is that, under projection, an observer underweights the true information of the previous generation. Moreover, they wrongly put weight on irrelevant factors (i.e., the price and their own taste), and this erroneous weight is larger when signals are less precise relative to the prior (i.e., when γ_1 is smaller). There is a straightforward intuition for this. A projector will, on average, observe a level of demand that deviates from their initial expectations since they incorrectly predict demand conditional on the signal. They attribute this deviation to the value of s_1. Thus, when a projector anticipates that the signal will have little effect on predecessors' beliefs (i.e.,. γ_1 is small), they require a more extreme value of s_1 to rationalize the deviation between the observed demand and their biased predictions.

Now consider demand in Generation 2. An agent with taste t forms an expectation of ω based on s_2 and $\hat{\omega}_1(t)$ equal to $\mathbb{E}[\omega|s_2, \hat{\omega}_1(t)] = \gamma_2 s_2 + (1-\gamma_2)\hat{\omega}_1(t)$. Using the expression for $\hat{\omega}_1(t)$ above, the expected valuation of an agent in Generation 2 with taste t is

$$\mathbb{E}[u(\omega,t)|s_2, \hat{\omega}_1(t)] = \gamma_2 s_2 + (1-\gamma_2)\Big((1-\alpha)\tilde{\omega}_1 + \alpha p_1\Big) + \Big(1 - \alpha(1-\gamma_2)\Big)t. \tag{A.12}$$

Let $\hat{v}_2(t)$ denote the expected valuation in (A.12). Similar to the approach in the main text, we can write this perceived valuation in terms of a taste-independent component, denoted by $\bar{\omega}_2$, where

$$\bar{\omega}_2 \equiv \gamma_2 s_2 + (1-\gamma_2)\bigg((1-\alpha)\tilde{\omega}_1 + \alpha p_1\bigg). \tag{A.13}$$

In the rational model (i.e., $\alpha = 0$), $\bar{\omega}_2$ reduces to $\tilde{\omega}_2$—the rational expectation of ω given (s_1, s_2). Given (A.13), we can write perceived valuations in Generation 2 as $\hat{v}_2(t) = \bar{\omega}_2 + \beta_2 t$, where $\beta_2 \equiv 1-\alpha(1-\gamma_2)$.

The Evolution of Beliefs. In fact, the perceived valuations of consumers in *all* Generations $n \geq 2$ can be expressed as $\hat{v}_n(t) = \bar{\omega}_n + \beta_n t$ where $\bar{\omega}_n$ is independent of tastes. Thus, the dynamics of the model are described by the evolution of the sequences of $(\bar{\omega}_n)$ and (β_n).

To verify for this claim, suppose that, as in Generation 2, the perceived valuations of agents in any Generation $n > 2$ are given by $\hat{v}_n(t) = \bar{\omega}_n + \beta_n t$. The demand in period $n \geq 2$ is then

$$D_n(p_n; \bar{\omega}_n) \equiv 1 - F\left(\frac{1}{\beta_n}(p_n - \bar{\omega}_n)\right). \tag{A.14}$$

A projecting agent in Generation $n+1$ with taste t thinks that agents in Generation n share a common expectation of ω, denoted $\hat{\omega}$, and thus have a demand given by

$$\widehat{D}_n(p_n; \hat{\omega}|t) = 1 - \widehat{F}(p_n - \hat{\omega}|t) = 1 - F\left(\frac{p_n - \hat{\omega} - \alpha t}{1-\alpha}\right). \tag{A.15}$$

The agent thus infers that the posterior expectation of Generation n is the value of $\hat{\omega}$ that equates (A.14) and (A.15), yielding

$$\hat{\omega}_n(t) = \left(\frac{1-\alpha}{\beta_n}\right)\bar{\omega}_n + \left(1 - \frac{1-\alpha}{\beta_n}\right)p_n - \alpha t. \tag{A.16}$$

Thus, the updated expectation of ω for an agent with taste t in Generation $n+1$ is

$$\mathbb{E}[\omega|s_{n+1}, \hat{\omega}_n(t)] = \gamma_{n+1}s_{n+1} + (1-\gamma_{n+1})\bigg[\left(\frac{1-\alpha}{\beta_n}\right)\bar{\omega}_n + \left(1 - \frac{1-\alpha}{\beta_n}\right)p_n - \alpha t\bigg]. \tag{A.17}$$

This agent's total perceived valuation is $\hat{v}_{n+1}(t) = \mathbb{E}[\omega|s_{n+1}, \hat{\omega}_n(t)] + t$; hence,

$$\hat{v}_{n+1}(t) = \underbrace{\gamma_{n+1}s_{n+1} + (1-\gamma_{n+1})\bigg[\left(\frac{1-\alpha}{\beta_n}\right)\bar{\omega}_n + \left(1 - \frac{1-\alpha}{\beta_n}\right)p_n\bigg]}_{\equiv \bar{\omega}_{n+1}} + \underbrace{\bigg(1 - \alpha(1-\gamma_{n+1})\bigg)}_{\equiv \beta_{n+1}} t.$$

This reveals how (β_n) and $(\bar{\omega}_n)$ evolve:

$$\beta_{n+1} = 1 - \alpha(1-\gamma_{n+1}), \tag{A.18}$$

$$\bar{\omega}_{n+1} = \gamma_{n+1}s_{n+1} + (1-\gamma_{n+1})\bigg[\left(\frac{1-\alpha}{\beta_n}\right)\bar{\omega}_n + \left(1 - \frac{1-\alpha}{\beta_n}\right)p_n\bigg]. \tag{A.19}$$

Thus, for all $n \geq 2$, the perceived valuations of consumers in period n are given by $\hat{v}_n(t) =$

$\bar{\omega}_n + \beta_n t$, where β_n and $\bar{\omega}_n$ follow the processes in (A.18) and (A.19), respectively, starting from the initial conditions of $\beta_1 = 1$ and $\bar{\omega}_1 = \tilde{\omega}_1 = \gamma_1 s_1$. Furthermore, the quantity demanded in each period n is given by $d_n = D_n(p_n; \bar{\omega}_n)$ as in (A.14).[42]

There are a few features of this process worth noting. First, since γ_n is monotonically decreasing in n with $\lim_{n\to\infty} \gamma_n = 0$, it follows that β_n monotonically decreases from 1 and converges to $1-\alpha$. Thus, in every period, a consumer's perceived valuation puts too little (yet positive) weight on his own taste. In the limit, this diminished weight is equal to $1-\alpha$. This is identical to our results in both the static and dynamic cases of our model in the main text. See, for instance, the discussion preceding Proposition 2.

Additionally, since $\beta_n \in (1-\alpha, 1)$ for all n, the term $(1-\alpha)/\beta_n$ that appears in the transition equation for $(\bar{\omega}_n)$ must take a value in $(0,1)$. Thus, the term in square brackets in Equation (A.19) is a convex combination of $\bar{\omega}_n$ and p_n, implying that the aggregate biased belief in each period n is strictly increasing in the price faced by the previous generation. Furthermore, the weight on $\bar{\omega}_n$ (i.e., $(1-\alpha)/\beta_n$) converges to 1 as $n \to \infty$, and thus the effect of the preceding price on current beliefs diminishes over time.

Finally, we can use Equation (A.19) to write the beliefs of the current generation in terms of the entire history of signals and prices. Toward that end, let $\lambda_n \equiv (1-\alpha)/\beta_n \in (0,1)$. For all $k = 1, 2, \ldots$ and all $n \geq k+2$, define $a_k^n = \prod_{j=k+1}^{n-1} \lambda_j$. We then have:

$$\bar{\omega}_n = \gamma_n s_n + (1-\alpha)\gamma_n \left(\frac{1}{\beta_{n-1}} s_{n-1} + \sum_{k=1}^{n-2} \frac{a_k^n}{\beta_k} s_k \right) + \alpha\gamma_n \left(\frac{1}{\beta_{n-1}} p_{n-1} + \sum_{k=2}^{n-2} \frac{a_k^n}{\beta_k} p_k + \frac{a_1^n}{\gamma_1} p_1 \right). \quad \text{(A.20)}$$

The key implications of this expression are that aggregate biased beliefs put too little weight on predecessors' signals and instead erroneously put positive weight on all past prices.

The next result summarizes some of the points above, emphasizing that the comparative statics in our baseline model of the main text continue to hold within this richer signal structure.

Proposition A.2 (Comparative Statics in the Quasi-Public-Signal Model)**.** *Consider the signal structure of Section A.2. Beliefs and valuations in each period n follow the process described in* (A.19) *so long as demand remains interior (i.e., $d_k \in (0,1)$ for all $k < n$). In this case, the perceived quality of each agent in each period $n \geq 2$ is decreasing in their private value and increasing in all previous prices.*

B Inference from a Monopolist's Price

In this appendix, we consider a simple way in which projection can distort inferences from prices. Specifically, we show how our basic comparative static on taste-dependent inference emerges when an observer infers ω from the price a monopolist offers to fully-informed consumers. Namely, an observer with a higher private taste infers a lower quality. High types overestimate the price a

[42] Note that the transition equations in (A.18) and (A.19) characterize the process in the case where the quantity demanded in each period prior to $n+1$ is interior (i.e., $d_k \in (0,1)$ for $k \leq n$).

monopolist would charge conditional on any level of quality, and hence they consequently underestimate ω conditional on the observed price. The opposite logic holds for low types.

Suppose a projecting observer with taste t thinks that that p maximizes $p[1-\widehat{F}(p-\hat{\omega}|t)]$ where $\hat{\omega}$ is the expected quality among informed consumers. Hence, this agent thinks p is the solution to

$$p = \frac{1-\widehat{F}(p-\hat{\omega}|t)}{\hat{f}(p-\hat{\omega}|t)} \equiv 1/\hat{h}(p-\hat{\omega}|t), \tag{B.1}$$

where $\hat{h}(x|t)$ denotes the perceived hazard rate of a projector with taste t. From (2), we have

$$\hat{h}(x|t) = \frac{1}{1-\alpha}\frac{f\left(\frac{x-\alpha t}{1-\alpha}\right)}{1-F\left(\frac{x-\alpha t}{1-\alpha}\right)} = \frac{1}{1-\alpha}h\left(\frac{x-\alpha t}{1-\alpha}\right), \tag{B.2}$$

where h is the hazard rate associated with F. Since h is increasing (because we assumed f is log-concave), $\hat{h}(x|t)$ is also increasing on type t's perceived support for all $t \in \mathcal{T}$. Furthermore, for a fixed x, (B.2) reveals that $\hat{h}(x|t)$ is decreasing in t, and hence perceived distributions exhibit strict Hazard-Rate Dominance (HRD) with respect to t; that is, for any $t > t'$ and any x interior to both $\mathcal{T}(t)$ and $\mathcal{T}(t')$, we have $\hat{h}(x|t) < \hat{h}(x|t')$.

Let $\hat{\omega}(t)$ be a projector's estimated value of ω. This is the value of $\hat{\omega}$ that solves (B.1), and thus

$$\hat{\omega}(t) = p - \hat{h}^{-1}(1/p|t). \tag{B.3}$$

Given that the family of perceived distributions satisfies HRD, $\hat{h}^{-1}(x|t) > \hat{h}^{-1}(x|t') \Leftrightarrow t > t'$, and hence $\hat{\omega}(t)$ is decreasing in t—higher types form more pessimistic estimates of ω.

C Proofs

Proof of Proposition 1. We prove this result for a more general utility structure than assumed in the main text. Here, we assume that each agent's valuation for the good is given by a utility function $u(\omega, t)$ that is strictly increasing and differentiable with respect to both variables and satisfies $\frac{\partial^2}{\partial\omega\partial t}u(\omega,t) > 0$ for all $t \in \mathcal{T}$ and $\omega \in \mathbb{R}$. For simplicity, we also assume u is linear in ω.[43] Our model of projection easily accommodates such a generalization: An agent with private value t believes the utility of any agent with taste t' is $\hat{u}(\omega, t'|t) = \alpha u(\omega, t) + (1-\alpha)u(\omega, t')$. This misperceived utility function then pins down type t's perceived distribution of valuations in each state ω. We begin by proving the following lemma.

Lemma C.1. *Consider any u satisfying the assumptions above, and suppose that (p, s) admits interior demand. For any $\lambda > 0$ and $\alpha \in [0, 1)$, there exists a unique steady-state equilibrium; in that equilibrium, the quantity demanded is equal to the quantity demanded in the full-information benchmark (i.e., $\lambda = 1$).*

[43] The intuitions from the proof generalize beyond this risk-neutral case. However, we assume risk neutrality so that, as in the main text, each agent's mean belief, $\hat{\omega}$, is a sufficient statistic for their behavior irrespective of further details on their posterior distribution over ω. Thus, as in the main text, uninformed agents here attempt to extract the mean belief of informed agents, $\bar{\omega}(s)$. The proof below holds without the linearity assumption when informed agents are perfectly informed. And an analogous argument would hold beyond the linear case so long as we impose a similar structure on $U(s, t)$—an agent's *expected* utility conditional on s and t.

Step 1: Inference rules. We first derive an uninformed agent's inference from the observed quantity demanded, d. Since we focus on symmetric strategies, it is sufficient to derive the inference rule of an arbitrary agent with taste t. Let $\widehat{D}(p;\hat{\omega}|t)$ denote this agent's conjectured demand among a population of agents who believe the expected value of ω is $\hat{\omega}$;

$$\begin{aligned}\widehat{D}(p;\hat{\omega}|t) = \Pr\left[\alpha u(\hat{\omega};t) + (1-\alpha)u(\hat{\omega};T) \geq p\right] = \Pr\left[u(\hat{\omega};T) \geq \frac{p-\alpha u(\hat{\omega};t)}{1-\alpha}\right] \\ = \Pr\left[T \geq t^*\left(\frac{p-\alpha u(\hat{\omega};t)}{1-\alpha};\hat{\omega}\right)\right] = 1 - F\left(t^*\left(\frac{p-\alpha u(\hat{\omega};t)}{1-\alpha};\hat{\omega}\right)\right),\end{aligned} \quad \text{(C.1)}$$

where $t^*(p;\hat{\omega})$ is the inverse of $u(\hat{\omega};t)$ w.r.t. t evaluated at $\hat{\omega}$ and p. That is, $t^*(p;\hat{\omega})$ is such that $u(\hat{\omega};t^*(p;\hat{\omega})) = p$ for all $p \geq 0$ and $\hat{\omega} \in \mathbb{R}$. Note that t^* is well defined given our assumptions on u. Furthermore, let $t_1^*(p;\hat{\omega})$ and $t_2^*(p;\hat{\omega})$ denote the partial derivative of t^* w.r.t. the first and second argument, respectively; our assumptions on u also imply that for all $p \geq 0$ and $\hat{\omega} \in \mathbb{R}$, we have $t_1^*(p;\hat{\omega}) > 0$ and $t_2^*(p;\hat{\omega}) < 0$.

The inference rule of an uninformed agent with taste t is then given by the function $\hat{\omega}(\cdot|t,p) : [0,1] \to \mathbb{R}$ such that for all $d \in (0,1)$, $\hat{\omega}(d|t,p)$ is equal to the unique value of $\hat{\omega}$ that solves

$$d = 1 - F\left(t^*\left(\frac{p-\alpha u(\hat{\omega};t)}{1-\alpha};\hat{\omega}\right)\right), \quad \text{(C.2)}$$

and $\hat{\omega}(d|t,p)$ represents the agent's perceived expected value of ω. An uninformed agent with taste t buys if d is such that $u\left(\hat{\omega}(d|t,p),t\right) \geq p$. The steady-state condition for the static equilibrium is then:

$$d = \lambda D^I(p;\bar{\omega}(s)) + (1-\lambda)\Pr\left[u\left(\hat{\omega}(d|T,p),T\right) \geq p\right]. \quad \text{(C.3)}$$

Under our solution concept, a projecting agent with taste t believes that all agents (i) follow the same inference rule as him; (ii) form an expectation of ω equal to $\hat{\omega}(d|t,p)$; and (iii) take their expected-utility-maximizing action given this expectation. He therefore believes that, in equilibrium, his inference rule allows him to perfectly extract the signal of the informed agents. To see this, note that an agent with taste t thinks that demand among the informed is

$$\widehat{D}(p;\bar{\omega}(s)|t) = 1 - F\left(t^*\left(\frac{p-\alpha u(\bar{\omega}(s);t)}{1-\alpha};\bar{\omega}(s)\right)\right), \quad \text{(C.4)}$$

and thinks that

$$\begin{aligned}\Pr\left[u\left(\hat{\omega}(d|T,p),T\right) \geq p\right] &= \Pr\left[u\left(\hat{\omega}(d|t,p),T\right) \geq p\right] \\ &= 1 - F\left(t^*\left(\frac{p-\alpha u(\hat{\omega}(d|t,p);t)}{1-\alpha};\hat{\omega}(d|t,p)\right)\right) = d\end{aligned} \quad \text{(C.5)}$$

where the third equality follows from the fact that, by definition, $\hat{\omega}(d|t,p)$ is the value of $\hat{\omega}$ that solves (C.2). Thus, substituting (C.4) and (C.5) into (C.3) reveals that the agent believes that, in equilibrium, the aggregate quantity demanded is such that

$$d = \lambda \left(1 - F\left(t^*\left(\frac{p - \alpha u(\bar{\omega}(s); t_i)}{1-\alpha}; \bar{\omega}(s)\right)\right)\right) + (1-\lambda)d$$
$$\Rightarrow \quad d = 1 - F\left(t^*\left(\frac{p - \alpha u(\bar{\omega}(s); t_i)}{1-\alpha}; \bar{\omega}(s)\right)\right). \quad \text{(C.6)}$$

Within this agent's model, both (C.5) and (C.6) must hold, and hence the agent believes

$$1 - F\left(t^*\left(\frac{p - \alpha u(\bar{\omega}(s); t)}{1-\alpha}; \bar{\omega}(s)\right)\right) = 1 - F\left(t^*\left(\frac{p - \alpha u(\hat{\omega}(d|t,p); t_i)}{1-\alpha}; \hat{\omega}(d|t,p)\right)\right), \quad \text{(C.7)}$$

which implies that $\hat{\omega}(d|t,p) = \bar{\omega}(s)$ since $\hat{\omega}(d|t,p)$ is the unique value of $\hat{\omega}$ that solves (C.2).

By this logic, this inference rule does perfectly reveal the informed agents' private information when all agents are rational (i.e., $\alpha = 0$), since in this case (C.7) reduces to $t^*(p; \bar{\omega}(s)) = t^*(p; \hat{\omega}(d|t,p))$ and thus in reality we have $\hat{\omega}(d|t,p) = \bar{\omega}(s)$ since t^* is strictly decreasing in $\hat{\omega}$.

Step 2: $\hat{\omega}(d|t,p)$ is strictly decreasing in t. Next, we show that $\hat{\omega}(d|t,p)$ is strictly decreasing in t. Recall that for any fixed $d \in (0,1)$, Condition (C.2) implies that $\hat{\omega}(d|t,p)$ solves

$$L(\hat{\omega}|t,p) \equiv t^*\left(\frac{p - \alpha u(\hat{\omega}; t)}{1-\alpha}; \hat{\omega}\right) - F^{-1}(1-d) = 0. \quad \text{(C.8)}$$

By the implicit function theorem (IFT), we have

$$\frac{\partial \hat{\omega}(d|t,p)}{\partial t} = -\left(\frac{\partial L(\hat{\omega}|t,p)}{\partial t}\right)\left(\frac{\partial L(\hat{\omega}|t,p)}{\partial \hat{\omega}}\right)^{-1}\Bigg|_{\hat{\omega}=\hat{\omega}(d|t,p)}. \quad \text{(C.9)}$$

Notice that

$$\frac{\partial L(\hat{\omega}|t,p)}{\partial t} = -t_1^*\left(\frac{p - \alpha u(\hat{\omega}; t)}{1-\alpha}; \hat{\omega}\right)\left(\frac{\alpha}{1-\alpha}\right)\frac{\partial u(\hat{\omega}; t)}{\partial t} < 0, \quad \text{(C.10)}$$

and

$$\frac{\partial L(\hat{\omega}|t,p)}{\partial \hat{\omega}} = -t_1^*\left(\frac{p - \alpha u(\hat{\omega}; t)}{1-\alpha}; \hat{\omega}\right)\left(\frac{\alpha}{1-\alpha}\right)\frac{\partial u(\hat{\omega}; t)}{\partial t} + t_2^*\left(\frac{p - \alpha u(\hat{\omega}; t)}{1-\alpha}; \hat{\omega}\right) < 0, \quad \text{(C.11)}$$

and hence (C.9) implies that $\frac{\partial \hat{\omega}(d|t,p)}{\partial t} < 0$.

Step 3: Total perceived valuations, $u(\hat{\omega}(d|t,p), t)$, are increasing in t. Although perceived quality is decreasing in t (Step 2), total perceived valuations remain increasing in t. Notice that

$$\frac{\partial u(\hat{\omega}(d|t,p), t)}{\partial t} = \frac{\partial u(\hat{\omega}(d|t,p); t)}{\partial \hat{\omega}}\frac{\partial \hat{\omega}(d|t,p)}{\partial t} + \frac{\partial u(\hat{\omega}(d|t,p); t)}{\partial t}, \quad \text{(C.12)}$$

and thus $\frac{\partial u(\hat{\omega}(d|t,p),t)}{\partial t} > 0$ iff

$$\frac{\partial \hat{\omega}(d|t,p)}{\partial t} > -\left(\frac{\partial u(\hat{\omega}; t)}{\partial t}\right)\left(\frac{\partial u(\hat{\omega}; t)}{\partial \hat{\omega}}\right)^{-1}\Bigg|_{\hat{\omega}=\hat{\omega}(d|t,p)}. \quad \text{(C.13)}$$

Substituting (C.10) and (C.11) into (C.9) implies that

$$\frac{\partial\hat{\omega}(d|t,p)}{\partial t} = -\left(\frac{\partial u(\hat{\omega};t)}{\partial t}\right)\left(\frac{\partial u(\hat{\omega};t)}{\partial\hat{\omega}} + K\right)^{-1}\Big|_{\hat{\omega}=\hat{\omega}(d|t,p)}, \tag{C.14}$$

where

$$K = -\left(\frac{1-\alpha}{\alpha}\right)\underbrace{t_2^*\left(\frac{p-\alpha u(\hat{\omega};t)}{1-\alpha};\hat{\omega}\right)}_{<0}\underbrace{\left(t_1^*\left(\frac{p-\alpha u(\hat{\omega};t)}{1-\alpha};\hat{\omega}\right)\right)^{-1}}_{>0}\Big|_{\hat{\omega}=\hat{\omega}(d|t,p)}, \tag{C.15}$$

and hence (C.13) holds given that $K \geq 0$. Note that K is strictly positive if $\alpha > 0$ and hence equilibrium total perceived valuations are strictly increasing in t under projection.

Step 4: The fraction of uninformed agents who buy follows a cutoff rule and is equal to fraction of informed agents who buy. The equilibrium condition in (C.3) depends on the fraction of uninformed agents who buy in the steady state, $\Pr[u(\hat{\omega}(d|T,p),T) \geq p]$. Since Step 3 ensures that $u(\hat{\omega}(d|t,p);t)$ is strictly increasing in t, there must exist a threshold value $\hat{t}(d)$ such that, in equilibrium, types with with $t \geq \hat{t}(d)$ buy and those with $t < \hat{t}(d)$ do not. That is, there is a well-defined "marginal uninformed type", $\hat{t}(d)$, that naturally separates the type space into buyers and non-buyers.

We now show that, for any value of $d \in (0,1)$, it must be that $\hat{t}(d) = F^{-1}(1-d)$. That is, the marginal uninformed type is such that the fraction of uninformed agents who buy is equal to d. To see this, the inference of an agent of any type t, $\hat{\omega}(d|t,p)$, must satisfy

$$u\left(\hat{\omega}(d|t,p); t^*\left(\frac{p-\alpha u(\hat{\omega}(d|t,p),t)}{1-\alpha};\hat{\omega}(d|t,p)\right)\right) = \frac{p-\alpha u(\hat{\omega}(d|t,p),t)}{1-\alpha}; \tag{C.16}$$

this follows from the fact that, by definition, $t^*(\tilde{u};\hat{\omega}(d|t,p))$ is the value of t such that $u(\hat{\omega}(d|t,p),t) = \tilde{u}$. Furthermore, recall that for all t, the inference rule $\hat{\omega}(d|t,p)$ is such that (C.8) holds as an identity; substituting this identity into (C.16) and rearranging implies that

$$p = \alpha u(\hat{\omega}(d|t,p);t) + (1-\alpha)u\big(\hat{\omega}(d|t,p);F^{-1}(1-d)\big). \tag{C.17}$$

Given that the condition above must hold for all $t \in \mathcal{T}$, it must hold for type $\hat{t}(d) \equiv F^{-1}(1-d)$ whose private value lies at the $(1-d)$-percentile in the taste distribution. Condition (C.17) evaluated at $\hat{t}(d) = F^{-1}(1-d)$ implies

$$\begin{aligned} p &= \alpha u\big(\hat{\omega}(d|\hat{t}(d),p);F^{-1}(1-d)\big) + (1-\alpha)u\big(\hat{\omega}(d|\hat{t}(d),p);F^{-1}(1-d)\big) \\ &= u\big(\hat{\omega}(d|\hat{t}(d),p);\hat{t}(d)\big). \end{aligned} \tag{C.18}$$

Thus, an agent with type $\hat{t}(d) = F^{-1}(1-d)$ forms an inference that leaves him indifferent between buying or not. By Step 3, above, we know that an agent with $t > \hat{t}(d)$ must form an inference such that he has a strict preference to buy, while one with $t < \hat{t}(d)$ must form an inference such that he has a strict preference to not buy. Thus $\hat{t}(d)$ represents the marginal uninformed type, and the fraction of uninformed agents who buy is thus $\Pr[u(\hat{\omega}(d|T,p),T) \geq p] = 1 - F\big(\hat{t}(d)\big) = 1 - F(F^{-1}(1-d)) = d$.

Step 5: The total fraction of agents who buy in equilibrium is equal to the fraction of informed agents who buy. Recall from (C.3) that, in equilibrium, the aggregate quantity demanded must satisfy

$$d = \lambda D^I(p; \bar{\omega}(s)) + (1-\lambda)\Pr\left[u\left(\hat{\omega}(d|T,p), T\right) \geq p\right]. \tag{C.19}$$

From Step 4, we know that $\Pr\left[u\left(\hat{\omega}(d|T,p), T\right) \geq p\right] = d$, and hence the equilibrium condition reduces to

$$d = \lambda D^I(p; \bar{\omega}(s)) + (1-\lambda)d \;\;\Rightarrow\;\; d = D^I(p; \bar{\omega}(s)). \tag{C.20}$$

This completes the proof of the lemma. We now establish each part of Proposition 1.

Part 1. Let $\hat{\omega}(t)$ denote the steady-state inference of an uninformed agent who has taste t; that is, $\hat{\omega}(t) \equiv \hat{\omega}(d^*|t,p)$, where $d^* \equiv D^I(p; \bar{\omega}(s))$ is the quantity demanded in equilibrium. The fact that $\hat{\omega}(t)$ is strictly decreasing in t is established in Step 2 in the proof of Lemma C.1.

Recall from Step 4 of Lemma C.1 that the marginal uninformed type is $\hat{t}(d) = F^{-1}(1-d)$. Since $d = D^I(p; \bar{\omega}(s)) = [1 - F(t^*(p; \bar{\omega}(s)))]$ in equilibrium, we therefore have $\hat{t}(d) = t^*(p; \bar{\omega}(s))$ in equilibrium. That is, the marginal uninformed type is equal to the marginal informed type. This further implies that the an uninformed agent with $t = t^*(p; \bar{\omega}(s))$ is the unique uninformed type who correctly estimates the state: substituting $\hat{t}(d) = t^*(p; \bar{\omega}(s))$ into (C.18) implies that this type forms an inference that leaves him indifferent between buying or not, which means that he must form the same expectation as the informed agent who is truly indifferent; hence, $\hat{\omega}(d|t^*(p; \bar{\omega}(s)), p) = \bar{\omega}(s)$ at the equilibrium value of d. Since $\hat{\omega}(t)$ is strictly decreasing in t, this implies that uninformed agents with $t > t^*(p; \bar{\omega}(s))$ underestimate the state, while those with $t < t^*(p; \bar{\omega}(s))$ overestimate the state.

Part 2. We know argue that $\hat{\omega}(t)$ is increasing in p for each $t \in \mathcal{T}$. Condition (C.2) implies that $\hat{\omega}(d|t,p)$ solves

$$L(\hat{\omega}|t,p) \equiv t^*\left(\frac{p - \alpha u(\hat{\omega}; t)}{1-\alpha}; \hat{\omega}\right) - F^{-1}(1-d) = 0. \tag{C.21}$$

In the steady-state, $d = D^I(p; \bar{\omega}(s)) = 1 - F(t^*(p; \bar{\omega}(s)))$ and hence $F^{-1}(1-d) = t^*(p; \bar{\omega}(s))$; the preceding condition implies that $\hat{\omega}(t)$ solves

$$L(\hat{\omega}|t,p) \equiv t^*\left(\frac{p - \alpha u(\hat{\omega}; t)}{1-\alpha}; \hat{\omega}\right) - t^*(p; \bar{\omega}(s)) = 0. \tag{C.22}$$

The IFT then implies

$$\frac{\partial \hat{\omega}(t)}{\partial p} = -\left(\frac{\partial L(\hat{\omega}|t,p)}{\partial p}\right)\left(\frac{\partial L(\hat{\omega}|t,p)}{\partial \hat{\omega}}\right)^{-1}\Bigg|_{\hat{\omega}=\hat{\omega}(t)}, \tag{C.23}$$

and (C.11) shows that $\frac{\partial L(\hat{\omega}|t,p)}{\partial \hat{\omega}} < 0$. Hence, $\frac{\partial \hat{\omega}(t)}{\partial p} > 0$ if and only if $\frac{\partial L(\hat{\omega}|t,p)}{\partial p}\Big|_{\hat{\omega}=\hat{\omega}(t)} > 0$. Notice that

$$\frac{\partial L(\hat{\omega}|t,p)}{\partial p} = t_1^*\left(\frac{p - \alpha u(\hat{\omega}; t)}{1-\alpha}; \hat{\omega}\right)\left(\frac{1}{1-\alpha}\right) - t_1^*(p; \bar{\omega}(s)). \tag{C.24}$$

We first show that (C.24) is positive at the margin; i.e., for type $t = t^*(p; \bar{\omega}(s))$. In this case, $\hat{\omega}(t) = \bar{\omega}(s)$ and thus $u(\hat{\omega}, t) = u(\bar{\omega}(s), t) = p$, implying that $t_1^*\left(\frac{p-\alpha u(\hat{\omega};t)}{1-\alpha}; \hat{\omega}\right) = t_1^*(p; \bar{\omega}(s))$. Hence, (C.24) is positive if and only if $\alpha > 0$. To see why this condition must hold more generally, let $\hat{\omega}(t|p)$ denote the equilibrium perception of an agent with taste t facing price p, and consider p_0 and $p_1 > p_0$. Let $t_0^* \equiv t^*(p_0; \bar{\omega}(s))$. The preceding argument establishes that $\hat{\omega}(t_0^*|p_1) > \hat{\omega}(t_0^*|p_0)$. Furthermore, from Part 1, we know that $\hat{\omega}(t|p)$ is strictly decreasing in t for each $p \in \{p_0, p_1\}$. Since $\hat{\omega}(t_0^*|p_1) > \hat{\omega}(t_0^*|p_0)$, we must have $\hat{\omega}(t|p_1) > \hat{\omega}(t|p_0)$ for all t if $\hat{\omega}(\cdot|p_0)$ and $\hat{\omega}(\cdot|p_1)$ do not cross; that is, if there exists no $\tilde{t} \in \mathcal{T}$ such that $\hat{\omega}(\tilde{t}|p_1) = \hat{\omega}(\tilde{t}|p_0)$. Toward a contradiction, suppose such a $\tilde{t}$ exists, and let $\tilde{\omega} = \hat{\omega}(\tilde{t}|p_1) = \hat{\omega}(\tilde{t}|p_0)$. By definition, $\tilde{\omega}$ must rationalize the observed levels of demand at prices p_0 and p_1. But this contradicts the fact that the agent must infer distinct estimates of ω from these different levels of demand. Moreover, it is immediate that (C.24) is strictly positive, as desired, for the functional form for u considered in the main text whenever $\alpha > 0$ since in this case t_1^* is a constant. ■

Proof of Proposition 2. We prove this result for the more general class of utility functions introduced at the beginning of the proof of Proposition 1 (i.e.,$u(\omega, t)$ is strictly increasing and differntiable w.r.t. both variables, satisfies $\frac{\partial^2}{\partial\omega\partial t}u(\omega, t) > 0$, and is linear in ω). Thus, the results of the generalized version of Proposition 1 apply.

The random variable describing valuations of the uninformed agents in the rational steady-state equilibrium is $v(T) \equiv u(\bar{\omega}(s), T)$. Under projection, this random variable is $\hat{v}(T) \equiv u(\hat{\omega}(T), T)$. We argue that $\hat{v}(\cdot)$ is a clockwise rotation of $v(\cdot)$. First, note that $\hat{v}(t^*) = u(\hat{\omega}(t^*), t^*) = u(\bar{\omega}(s), t^*) = v(t^*)$, which follows from the proof of Part 1 of Proposition 1 where we show that $\hat{\omega}(t^*) = \bar{\omega}(s)$. Thus, v and $\hat{v}$ intersect at t^*. Next, for $t > t^*$, $\hat{v}(t) = u(\hat{\omega}(t), t) < u(\bar{\omega}(s), t) = v(t)$ since $\hat{\omega}(t) < \bar{\omega}(s)$ for $t > t^*$ given that $\hat{\omega}(t)$ is strictly decreasing in t (as shown in Part 1 of Proposition 1). Similarly, for $t < t^*$, $\hat{v}(t) = u(\hat{\omega}(t), t) > u(\bar{\omega}(s), t) = v(t)$ since $\hat{\omega}(t) > \bar{\omega}(s)$ for $t < t^*$, which again follows from $\hat{\omega}(t)$ being strictly decreasing in t. Thus, $\hat{v}$ is clockwise rotation of v. Since v and $\hat{v}$ are both strictly increasing functions, this rotation property implies that $\hat{v}(T)$ is less disperse than $v(T)$ in the sense of the dispersion order defined by Shaked and Shanthikumar (2007); i.e., $\hat{v}(T) \leq_{disp} v(T)$ (see the end of the proof of Proposition A.1 in Appendix A.1 for the definition of this order). Thus, by Theorem 3.B.16 of Shaked and Shanthikumar (2007), $\text{Var}\big(\hat{v}(T)\big) < \text{Var}\big(v(T)\big)$. ■

Proof of Lemma 1. We will prove the claim by induction on $n = 2, \dots, N$. As argued in the main text preceding Equation (11), $\hat{\omega}_2(t) = \bar{\omega}_2 - \alpha t$ for some $\bar{\omega}_2$ independent of t. This establishes the base case. Now suppose that in period n, $\hat{\omega}_n(t) = \bar{\omega}_n - \alpha t$. The marginal uninformed type in period n has taste $\hat{t}_n$ such that $\hat{\omega}_n(\hat{t}_n) + \hat{t}_n = p_n \Rightarrow \hat{t}_n = (p_n - \bar{\omega}_n)/(1-\alpha)$ and thus aggregate demand in period n is $d_n = \lambda\big[1 - F(p_n - \bar{\omega}(s))\big] + (1-\lambda)\left[1 - F\left(\frac{p_n - \bar{\omega}_n}{1-\alpha}\right)\right]$. An observer in generation $n+1$ then forms a perception of ω equal to $\hat{\omega}_{n+1}(t)$ such that $d_n = 1 - \widehat{F}(p_n - \hat{\omega}_{n+1}(t)) = 1 - F(\frac{p_n - \hat{\omega}_{n+1}(t) - \alpha t}{1-\alpha}) \Rightarrow \hat{\omega}_{n+1}(t) = [p_n - (1-\alpha)F^{-1}(1-d_n)] - \alpha t = \bar{\omega}_{n+1} - \alpha t$, where $\bar{\omega}_{n+1} = p_n - (1-\alpha)F^{-1}(1-d_n)$ is independent of t. ■

Proof of Proposition 3. The statement is readily verified for $n = 2$. For $n \geq 3$, the proof proceeds in two steps. First, we will show that $\overline{\omega}_n$ is increasing in p_{n-1}. Second, we will show that $\overline{\omega}_n$ is increasing in $\overline{\omega}_{n-1}$. Combining these two results then implies that $\overline{\omega}_n$ is increasing in p_k for all $n \geq 3$ and $k < n$.

Step 1: $\overline{\omega}_n$ *is increasing in* p_{n-1}. From Equation (14), $\overline{\omega}_n$ is the unique value that solves

$$\begin{aligned}\widehat{D}\left(p_{n-1};\overline{\omega}_n\right) &= D\left(p_{n-1};\overline{\omega}_{n-1},\overline{\omega}\left(s\right)\right)\Leftrightarrow\\ 1-F\left(\frac{p_{n-1}-\overline{\omega}_n}{1-\alpha}\right) &= \lambda\left[1-F\left(p_{n-1}-\overline{\omega}\left(s\right)\right)\right]+\left(1-\lambda\right)\left[1-F\left(\frac{p_{n-1}-\overline{\omega}_{n-1}}{1-\alpha}\right)\right].\end{aligned}$$

Solving this expression for $\bar{\omega}_n$ yields

$$\overline{\omega}_n = p_{n-1}-\left(1-\alpha\right)F^{-1}\left(\lambda F\left(p_{n-1}-\overline{\omega}\left(s\right)\right)+\left(1-\lambda\right)F\left(\frac{p_{n-1}-\overline{\omega}_{n-1}}{1-\alpha}\right)\right). \tag{C.25}$$

Differentiating (C.25) with respect to p_{n-1} then yields

$$\frac{\partial\overline{\omega}_n}{\partial p_{n-1}} = 1-\left(1-\alpha\right)\left[\frac{\lambda f\left(p_{n-1}-\overline{\omega}\left(s\right)\right)+\left(\frac{1-\lambda}{1-\alpha}\right)f\left(\frac{p_{n-1}-\overline{\omega}_{n-1}}{1-\alpha}\right)}{f\left(F^{-1}\left(\lambda F\left(p_{n-1}-\overline{\omega}\left(s\right)\right)+\left(1-\lambda\right)F\left(\frac{p_{n-1}-\overline{\omega}_{n-1}}{1-\alpha}\right)\right)\right)}\right]. \tag{C.26}$$

Let $t^*_{n-1} := p_{n-1}-\overline{\omega}\left(s\right)$ and $\hat{t}_{n-1} := \frac{p_{n-1}-\overline{\omega}_{n-1}}{1-\alpha}$; these denote the type of the marginal informed and uninformed buyer in period $n-1$, respectively. Also let

$$\phi_{n-1}\left(\lambda\right)\equiv f\left(F^{-1}\left(\lambda F\left(t^*_{n-1}\right)+\left(1-\lambda\right)F\left(\hat{t}_{n-1}\right)\right)\right)-\left(1-\alpha\right)\lambda f\left(t^*_{n-1}\right)-\left(1-\lambda\right)f\left(\hat{t}_{n-1}\right).$$

Then Equation C.26 implies that $\frac{\partial\overline{\omega}_n}{\partial p_{n-1}}>0\Leftrightarrow\ \phi_{n-1}\left(\lambda\right)>0$.

Next, we show that the function $\phi_{n-1}\left(\lambda\right)$ is positive for any $\lambda\in[0,1]$. Begin by noticing that $\phi_{n-1}\left(0\right)=0<\alpha f\left(t^*_{n-1}\right)=\phi_{n-1}\left(1\right)$. Moreover, we have

$$\begin{aligned}\phi'_{n-1}\left(\lambda\right)=\frac{f'\left(F^{-1}\left(\lambda F\left(t^*_{n-1}\right)+\left(1-\lambda\right)F\left(\hat{t}_{n-1}\right)\right)\right)}{f\left(F^{-1}\left(\lambda F\left(t^*_{n-1}\right)+\left(1-\lambda\right)F\left(\hat{t}_{n-1}\right)\right)\right)}\left[F\left(t^*_{n-1}\right)-F\left(\hat{t}_{n-1}\right)\right]\\ -\left(1-\alpha\right)f\left(t^*_{n-1}\right)+f\left(\hat{t}_{n-1}\right).\end{aligned} \tag{C.27}$$

Notice that neither the sign of $-\left(1-\alpha\right)f\left(t^*_{n-1}\right)+f\left(\hat{t}_{n-1}\right)$ nor that of $F\left(t^*_{n-1}\right)-F\left(\hat{t}_{n-1}\right)$ depend on λ. Moreover, since f is log-concave, the ratio $\frac{f'}{f}$ is decreasing. Hence, $\phi'_{n-1}\left(\lambda\right)$ can change sign at most once. Additionally, we have

$$\phi'_{n-1}\left(\lambda\right)\Big|_{\lambda=0}=\frac{f'\left(\hat{t}_{n-1}\right)}{f\left(\hat{t}_{n-1}\right)}\left[F\left(t^*_{n-1}\right)-F\left(\hat{t}_{n-1}\right)\right]-\left(1-\alpha\right)f\left(t^*_{n-1}\right)+f\left(\hat{t}_{n-1}\right)$$

and

$$\phi'_{n-1}\left(\lambda\right)\Big|_{\lambda=1}=\frac{f'\left(t^*_{n-1}\right)}{f\left(t^*_{n-1}\right)}\left[F\left(t^*_{n-1}\right)-F\left(\hat{t}_{n-1}\right)\right]-\left(1-\alpha\right)f\left(t^*_{n-1}\right)+f\left(\hat{t}_{n-1}\right).$$

Hence,

$$\phi'_{n-1}\left(\lambda\right)\Big|_{\lambda=0}-\phi'_{n-1}\left(\lambda\right)\Big|_{\lambda=1}=\left[\frac{f'\left(\hat{t}_{n-1}\right)}{f\left(\hat{t}_{n-1}\right)}-\frac{f'\left(t^*_{n-1}\right)}{f\left(t^*_{n-1}\right)}\right]\left[F\left(t^*_{n-1}\right)-F\left(\hat{t}_{n-1}\right)\right].$$

The previous expression is strictly positive since F is increasing while $\frac{f'}{f}$ is decreasing. Hence,

$\phi'_{n-1}(\lambda)\big|_{\lambda=0} > \phi'_{n-1}(\lambda)\big|_{\lambda=1}$. Summing up: (i) $\phi_{n-1}(\lambda)$ starts at zero (i.e., $\phi_{n-1}(0) = 0$) and ends positive (i.e., $\phi_{n-1}(1) = \alpha f(t_i) > 0$); (ii) for $\lambda \in (0,1)$, $\phi'_{n-1}(\lambda)$ can change sign at most once; (iii) $\phi'_{n-1}(\lambda)$ is greater at $\lambda = 0$ than at $\lambda = 1$. Hence, $\phi_{n-1}(\lambda) > 0$ for any $\lambda \in (0,1]$. Thus, $\frac{\partial \overline{\omega}_n}{\partial p_{n-1}} > 0$.

Step 2: *$\overline{\omega}_n$ is increasing in $\overline{\omega}_{n-1}$*. Differentiating (C.25) with respect to $\overline{\omega}_{n-1}$ yields

$$\frac{\partial \overline{\omega}_n}{\partial \overline{\omega}_{n-1}} = \frac{(1-\lambda) f\left(\frac{p_{n-1}-\overline{\omega}_{n-1}}{1-\alpha}\right)}{f\left(F^{-1}\left(\lambda F\left(p_{n-1}-\overline{\omega}(s)\right) + (1-\lambda) F\left(\frac{p_{n-1}-\overline{\omega}_{n-1}}{1-\alpha}\right)\right)\right)}.$$

The expression above is clearly positive for any $\lambda \in (0,1]$. Hence, $\overline{\omega}_n$ is increasing in $\overline{\omega}_{n-1}$. ■

Proof of Proposition 4. *Part 1: Initial Overreaction.* We will focus on the case with $\tilde{p} < p$; the case with $\tilde{p} > p$ is analogous and thus omitted.

Step 1: Quantity demanded is constant prior to the price change. Suppose $n^* \geq 2$. For ease of exposition, let $d^I \equiv D^I(p;\bar{\omega}(s))$ and $\tilde{d}^I \equiv D^I(\tilde{p};\bar{\omega}(s))$ denote the fraction of informed agents who buy at p and $\tilde{p}$, respectively. In period 1, $d_1 = D^I(p;\bar{\omega}(s)) = d^I$. The aggregate biased belief in period 2 is $\bar{\omega}_2 = (1-\alpha)\bar{\omega}(s) + \alpha p$, and Equation (11) then implies that the fraction of uninformed agents who buy in period 2 is $D^U(p;\bar{\omega}_2) = d_I$. Thus, the overall fraction of agents who buy in period 2 is $d_2 = d^I$. Equation (14) then implies that $\bar{\omega}_3 = \bar{\omega}_2$. Hence, if $n^* \geq 3$, then $d_3 = d_2 = d^I$. It is straightforward to see that this logic giving rise to a constant aggregate biased belief and quantity demanded will continue until the first period with the new price, $\tilde{p}$.

Step 2: Quantity demanded increases beyond the rational benchmark when the price drops. Since the quantity demanded is constant prior to the price change, we can (without loss of generality) assume from now on that $n^* = 1$. That is, $p_1 = p$ and $p_n = \tilde{p}$ for all $n \geq 2$. In all periods $n \geq 2$, the fraction of informed agents who buy is $\tilde{d}^I$. By contrast, in period 2, the fraction of uninformed agents who buy is $\tilde{d}^U_2 \equiv D^U(\tilde{p};\bar{\omega}_2) = 1 - F\left(\frac{\tilde{p}-\bar{\omega}_2}{1-\alpha}\right)$. Importantly, $\tilde{d}^U_2 > \tilde{d}^I$. To see this, note that $\bar{\omega}_2 = (1-\alpha)\bar{\omega}(s) + \alpha p$ and hence

$$\begin{aligned} \tilde{d}^U_2 &= 1 - F\left(\frac{\tilde{p} - (1-\alpha)\bar{\omega}(s) - \alpha p}{1-\alpha}\right) \\ &= 1 - F\left(\tilde{p} - \bar{\omega}(s) - \frac{\alpha}{1-\alpha}(p - \tilde{p})\right) > 1 - F(\tilde{p} - \bar{\omega}(s)) = \tilde{d}^I, \end{aligned} \tag{C.28}$$

where the inequality follows from $p - \tilde{p} > 0$. Thus, the total quantity demanded in period 2 is $d_2 = \lambda \tilde{d}^I + (1-\lambda)\tilde{d}^U_2$, which exceeds the rational benchmark of $\tilde{d}^I$.

Step 3: Quantity demanded remains above the rational benchmark in all subsequent periods. We now consider the path of $\tilde{d}^U_n \equiv D^U(\tilde{p};\bar{\omega}_n) = 1 - F\left(\frac{\tilde{p}-\bar{\omega}_n}{1-\alpha}\right)$ for $n > 2$ starting from the initial condition of $\tilde{d}^U_2 = 1 - F\left(\frac{\tilde{p}-\bar{\omega}_2}{1-\alpha}\right)$. From the law of motion in Equation (14), we must have that for all $n \geq 2$,

$$\tilde{d}^U_{n+1} = D^U(\tilde{p};\bar{\omega}_{n+1}) = \lambda \tilde{d}_I + (1-\lambda)\tilde{d}^U_n. \tag{C.29}$$

Thus, if $\tilde{d}^U_n > \tilde{d}^I$, then $\tilde{d}^U_{n+1} > \tilde{d}^I$. Since we start from the base case of $\tilde{d}^U_2 > \tilde{d}^I$, induction on n implies that $\tilde{d}^U_n > \tilde{d}^I$ for all $n \geq 2$. Thus, the aggregate quantity demanded in any period $n \geq 2$ is $d_n = \lambda \tilde{d}^I + (1-\lambda)\tilde{d}^U_n > \tilde{d}^I$, and d_n therefore exceeds the rational benchmark.

Part 2. We now show that d_n converges to the rational benchmark level of $\tilde{d}^I$ as $n \to \infty$. Toward this end, we first show that for all $k \geq 1$,

$$\tilde{d}^U_{k+2} = \left[1 - (1-\lambda)^k\right]\tilde{d}^I + (1-\lambda)^k \tilde{d}^U_2. \tag{C.30}$$

We will show by induction that in each period $k+2$, we have $\tilde{d}^U_{k+2} = a_{k+2}\tilde{d}^I + b_{k+2}\tilde{d}^U_2$, and that the coefficients a_{k+2} and b_{k+2} satisfy $a_{k+2} + b_{k+2} = 1$ and $b_{k+2} = (1-\lambda)^k$. The base case ($k = 1$) is immediate from (C.29), since $\tilde{d}^U_3 = \lambda \tilde{d}_I + (1-\lambda)\tilde{d}^U_2$ For the induction step, suppose the claim is true for $k > 1$. Thus, $\tilde{d}^U_{k+2} = a_{k+2}\tilde{d}^I + b_{k+2}\tilde{d}^U_2$. From (C.29), this implies that

$$\tilde{d}^U_{k+3} = \lambda\tilde{d}^I + (1-\lambda)[a_{k+2}D^I + b_{k+2}\tilde{d}^U_2] = \underbrace{[\lambda + (1-\lambda)a_{k+2}]}_{\equiv a_{k+3}}\tilde{d}^I + \underbrace{(1-\lambda)b_{k+2}}_{\equiv b_{k+3}}\tilde{d}^U_2. \tag{C.31}$$

It is then immediate that $b_{k+3} = (1-\lambda)^{k+1}$ as required given the induction assumption of $b_{k+2} = (1-\lambda)^k$. To show that $a_{k+3} + b_{k+3} = 1$, note that $a_{k+2} + b_{k+2} = 1$ implies

$$a_{k+3} + b_{k+3} = \lambda + (1-\lambda)a_{k+2} + (1-\lambda)b_{k+2} = \lambda + (1-\lambda)[a_{k+2} + b_{k+2}] = 1. \tag{C.32}$$

The deviation between the quantity demanded in period n under projection and the rational benchmark quantity is $|d_n - \tilde{d}^I| = |\lambda\tilde{d}^I + (1-\lambda)\tilde{d}^U_n - \tilde{d}^I| = (1-\lambda)|\tilde{d}^U_n - \tilde{d}^I|$, and (C.30) implies that for $n \geq 2$, $|\tilde{d}^U_n - \tilde{d}^I| = (1-\lambda)^{n-2}|\tilde{d}^U_2 - \tilde{d}^I|$. Thus,

$$|d_n - \tilde{d}^I| = (1-\lambda)^{n-1}|\tilde{d}^U_2 - \tilde{d}^I|. \tag{C.33}$$

This value is clearly decreasing in n and converges to 0 as $n \to \infty$. Thus, d_n converges to the rational benchmark, $\tilde{d}^I$, as $n \to \infty$. ■

Proof of Proposition 5. *Part 1.* The seller's objective is

$$\max_{p_1,p_2} \Pi(p_1, p_2; \alpha, \lambda), \tag{C.34}$$

subject to the dynamic constraint $\bar{\omega}_2 = \alpha p_1 + (1-\alpha)\bar{\omega}(s)$. Note that

$$\Pi(p_1, p_2; \alpha, \lambda) = p_1 D_1(p_1; \bar{\omega}(s)) + p_2 D_2(p_2; \bar{\omega}_2, \bar{\omega}(s)), \tag{C.35}$$

where, from Equation (12), we have

$$D_1(p_1; \bar{\omega}(s)) = D^I(p; \bar{\omega}(s)), \tag{C.36}$$
$$D_2(p_2; \bar{\omega}_2, \bar{\omega}(s)) = \lambda D^I(p_2; \bar{\omega}(s)) + (1-\lambda)D^U(p_2; \bar{\omega}_2), \tag{C.37}$$

with $D^I(p; \bar{\omega}(s)) = 1 - F(p - \bar{\omega}(s))$ and $D^U(p; \bar{\omega}_2) = 1 - F\left(\frac{p-\bar{\omega}_2}{1-\alpha}\right)$.

Potential Cases and Outline. We first describe the potential mix of interior and corner solutions and argue which of these are possible at the optimum. Then, for each possible case, we proceed to show that $p_1^* > p^M$ and $p_1^* > p_2^*$.

Fixing s, let $\underline{v} = \bar{\omega}(s) + \underline{t}$ and $\overline{v} = \bar{\omega}(s) + \bar{t}$ denote the expected valuations of the lowest and highest informed types, respective. The set of valuations among informed types is thus $\mathcal{V} = [\underline{v}, \overline{v}]$. As a function of p_1, an uninformed consumer's valuation in period 2 is $(1-\alpha)(\bar{\omega}(s) + t) + \alpha p_1$. Notice that at any optimum, $p_1 \in [\underline{v}, \bar{p}]$, where, recall, the price ceiling is $\bar{p} = \overline{v} - \kappa$ for some $\kappa > 0$

arbitrarily small such that $\bar{p} > p^M$. Hence, given p_1 and $\alpha > 0$, the set of valuations of uninformed consumers in period 2, denoted $\widehat{\mathcal{V}} \equiv [(1-\alpha)\underline{v} + \alpha p_1, (1-\alpha)\overline{v} + \alpha p_1]$, is a strict subset of $\mathcal{V}$.

First, notice that it is never optimal for the seller to serve all consumers in period 1. Since (p^M, s) admits interior demand, it is not optimal to serve all consumers in the rational benchmark; moreover, doing so under projection leads to the least attractive distribution of perceived valuations in period 2. Hence, in period 1 we either have an interior solution or a price equal to the price ceiling: $p_1^* \in (\underline{v}, \bar{p}]$.

Now consider possible corner cases in period 2. Since the valuations of uninformed types are a strict subset of the valuations of informed types, demand in period 2 is $D_2(p; \bar{\omega}_2; \bar{\omega}(s)) =$

$$\begin{cases} \lambda D^I(p; \bar{\omega}(s)) + (1-\lambda) & \text{if} \quad p \in [\underline{v}, (1-\alpha)\underline{v} + \alpha p_1), \\ \lambda D^I(p; \bar{\omega}(s)) + (1-\lambda) D^U(p; \bar{\omega}_2) & \text{if} \quad p \in [(1-\alpha)\underline{v} + \alpha p_1, (1-\alpha)\overline{v} + \alpha p_1], \\ \lambda D^I(p; \bar{\omega}(s)) & \text{if} \quad p \in ((1-\alpha)\overline{v} + \alpha p_1, \bar{p}]. \end{cases} \tag{C.38}$$

We now argue that the seller will never operate strictly within the first or third region of the demand function above, but may operate at the corner $p_2^c \equiv (1-\alpha)\underline{v} + \alpha p_1$ at which all uninformed types are served. First consider the third region. It is clearly sub-optimal to serve only informed types in period 2 since the strategy $p_1 = p_2 = p^M$ yields the seller the rational static monopoly profit in each period. Thus, deviating from these prices would require the seller to strictly benefit by serving consumers with manipulated beliefs, which is not possible when serving only informed types. Now consider the interior of the first region, where the seller sets a price below the lowest perceived valuation of uninformed types. This cannot happen at the optimum since it involves using p_1 to inflate the beliefs of uninformed types to an inefficient extent: since all uninformed types strictly prefer to buy at p_2 given $\bar{\omega}_2$, a slight reduction in p_1 would have no effect on the demand of the uninformed (or informed) agents in period 2 but would strictly increase the seller's profit in period 1. Thus, $p_2^* \geq p_2^c$ and in period 2 we either have an interior solution (in the middle region of C.38) or the corner solution such that $p_2^* = p_2^c$.

We now show that $p_1^* > p^M$ and $p_1^* > p_2^*$ in any of the possible cases noted above (i.e., interior or ceiling in period 1, and interior or corner in period 2).

Case 1: Interior Solutions. Substituting the dynamic constraint into D_2 in (C.36), the first-order conditions of C.34 are:

$$\frac{\partial}{\partial p_1} p_1 D_1(p_1; \bar{\omega}(s)) + p_2 \frac{\partial}{\partial \bar{\omega}_2} D_2(p_2; \bar{\omega}_2, \bar{\omega}(s)) \frac{\partial \bar{\omega}_2}{\partial p_1} = 0, \tag{C.39}$$

$$\frac{\partial}{\partial p_2} p_2 D_2(p_2; \bar{\omega}_2, \bar{\omega}(s)) = 0. \tag{C.40}$$

Define the following functions, each corresponding to the price derivative of the seller's profit in period n w.r.t. p_n for $n = 1, 2$:

$$M_1(p; \bar{\omega}(s)) \equiv \frac{\partial}{\partial p} p D_1(p; \bar{\omega}(s)), \tag{C.41}$$

$$M_2(p; \bar{\omega}_2, \bar{\omega}(s)) \equiv \frac{\partial}{\partial p} p D_2(p; \bar{\omega}_2, \bar{\omega}(s)). \tag{C.42}$$

Substituting these expressions along with the relevant derivatives into the FOCs above yields:

$$
\begin{aligned}
M_1(p_1;\bar{\omega}(s)) + p_2\left(\frac{\alpha(1-\lambda)}{1-\alpha}\right) f\left(\frac{p_2-\bar{\omega}_2}{1-\alpha}\right) &= 0, && \text{(C.43)}\\
M_2(p_2;\bar{\omega}_2,\bar{\omega}(s)) &= 0. && \text{(C.44)}
\end{aligned}
$$

Step 1: $p_1^* > p^M$*.* Since (p^M, s) admits interior demand under rational inference and since f being log-concave implies that F has an increasing hazard rate, M_1 is strictly decreasing in p and has exactly one root at $p^M > 0$. Note that FOC (C.43) implies that p_1^* solves

$$M_1(p_1^*;\bar{\omega}(s)) = -p_2^*\left(\frac{\alpha(1-\lambda)}{1-\alpha}\right) f\left(\frac{p_2^*-\bar{\omega}_2}{1-\alpha}\right), \tag{C.45}$$

where the right-hand side is strictly negative at an interior solution whenever $\alpha > 0$. Thus, since M_1 is decreasing in p and $M_1(p^M;\bar{\omega}(s)) = 0$, we must have $p_1^* > p^M$.

Step 2: $p_2^* < p_1^*$*.* FOC (C.44) implies that p_2^* solves $M_2(p_2^*;\bar{\omega}_2,\bar{\omega}(s)) = 0$. Toward a contradiction, suppose that $p_2^* = p_1^*$. We argue that $M_2(p_1^*;\bar{\omega}_2,\bar{\omega}(s)) < M_1(p_1^*;\bar{\omega}(s))$. Note that

$$M_2(p;\bar{\omega}_2,\bar{\omega}(s)) = D_2(p;\bar{\omega}_2,\bar{\omega}(s)) - p\left[\lambda f(p-\bar{\omega}(s)) + \left(\frac{1-\lambda}{1-\alpha}\right) f\left(\frac{p-\bar{\omega}_2}{1-\alpha}\right)\right]. \tag{C.46}$$

At $p = p_1^*$, we have $\bar{\omega}_2 = (1-\alpha)\bar{\omega}(s) + \alpha p_1^*$ and $(p-\bar{\omega}_2)/(1-\alpha) = p_1^* - \bar{\omega}(s)$, which further implies that $D_2(p_1^*;\bar{\omega}_2,\bar{\omega}(s)) = 1 - F(p_1^* - \bar{\omega}(s)) = D_1(p_1^*;\bar{\omega}(s))$. Thus, evaluating M_2 at $p = p_1^*$ yields

$$M_2(p_1^*;\bar{\omega}_2,\bar{\omega}(s)) = D_1(p_1^*;\bar{\omega}(s)) - p_1^* f(p_1^*-\bar{\omega}(s))\left(\frac{1-\alpha\lambda}{1-\alpha}\right). \tag{C.47}$$

However, note that $M_1(p_1^*;\bar{\omega}(s)) = D_1(p_1^*;\bar{\omega}(s)) - p_1^* f(p_1^*-\bar{\omega}(s))$, and thus $M_2(p_1^*;\bar{\omega}_2,\bar{\omega}(s)) < M_1(p_1^*;\bar{\omega}(s)) \;\Leftrightarrow$

$$-\,p_1^* f(p_1^*-\bar{\omega}(s))\left(\frac{1-\alpha\lambda}{1-\alpha}\right) < -p_1^* f(p_1^*-\bar{\omega}(s)), \tag{C.48}$$

which holds for any $\alpha > 0$. However, this presents a contradiction: since $M_1(p_1^*;\bar{\omega}(s)) < 0$ by FOC (C.43), $M_2(p_1^*;\bar{\omega}_2,\bar{\omega}(s)) < M_1(p_1^*;\bar{\omega}(s)) \Rightarrow M_2(p_1^*;\bar{\omega}_2,\bar{\omega}(s)) < 0$, which violates FOC (C.44). Thus, if $M_2(p;\bar{\omega}_2,\bar{\omega}(s))$ is decreasing in p, we must have $p_2^* < p_1^*$ in order for both FOCs to hold. To complete the proof, we only need to show that $M_2(p;\bar{\omega}_2,\bar{\omega}(s))$ is decreasing in p.

Step 3: M_2 *is decreasing in* p*.* Notice that

$$
\begin{aligned}
M_2(p;\bar{\omega}_2,\bar{\omega}(s)) &= \lambda \underbrace{\left[\frac{\partial}{\partial p} pD^I(p;\bar{\omega}(s))\right]}_{\equiv M^I(p;\bar{\omega}(s))} + (1-\lambda)\underbrace{\left[\frac{\partial}{\partial p} pD^U(p;\bar{\omega}_2)\right]}_{\equiv M^U(p;\bar{\omega}_2)}\\
&= \lambda M^I(p;\bar{\omega}(s)) + (1-\lambda)M^U(p;\bar{\omega}_2). && \text{(C.49)}
\end{aligned}
$$

It is immediate that $M^I(p;\bar{\omega}(s)) = M_1(p;\bar{\omega}(s))$ and is hence decreasing in p. Moreover, we can show that M^U is also decreasing in p given our assumptions on F. The following Lemma establishes this.

Lemma C.2. *Suppose the family of distributions* $\{F(x-\bar{\omega})\}_{\bar{\omega}\in\mathbb{R}}$ *is such that for any* $\bar{\omega}(s)$*,*

$M^I(p;\bar{\omega}(s)) \equiv \frac{\partial}{\partial p}p[1-F(p-\bar{\omega}(s))]$ *is decreasing at all* p *such that* $F(p-\bar{\omega}(s)) \in (0,1)$. *Then for any* $\alpha \in [0,1)$ *and* $\bar{\omega}_2 \in \mathbb{R}$, $M^U(p;\bar{\omega}_2) \equiv \frac{\partial}{\partial p}p[1-F(\frac{p-\bar{\omega}_2}{1-\alpha})]$ *is decreasing at all* p *such that* $F(\frac{p-\bar{\omega}_2}{1-\alpha}) \in (0,1)$.

We now prove Lemma C.2. Consider an arbitrary value of $\bar{\omega}(s) \in \mathbb{R}$. Notice that $M^I(p;\bar{\omega}(s)) = 1-F(p-\bar{\omega}(s))-pf(p-\bar{\omega}(s))$, and hence the assumption of the lemma implies $\frac{\partial}{\partial p}M^I(p;\bar{\omega}(s)) < 0 \Leftrightarrow -f(p-\bar{\omega}(s))-f(p-\bar{\omega}(s))-pf'(p-\bar{\omega}(s))$ on the relevant domain, which is equivalent to

$$-2f(p-\bar{\omega}(s))-pf'(p-\bar{\omega}(s)) \leq 0 \tag{C.50}$$

for all $\bar{\omega}(s)$ (and strictly so for $p-\bar{\omega}(s)$ on the interior of the support of F). Now note that $M^U(p;\bar{\omega}_2) \equiv \frac{\partial}{\partial p}p[1-F(\frac{p-\bar{\omega}_2}{1-\alpha})] = 1-F(\frac{p-\bar{\omega}_2}{1-\alpha})-pf(\frac{p-\bar{\omega}_2}{1-\alpha})\frac{1}{1-\alpha}$. To show that $M^U(p;\bar{\omega}(s))$ is decreasing in p, note that

$$\begin{aligned}\frac{\partial}{\partial p}M^U(p;\bar{\omega}_2) &= -f\left(\frac{p-\bar{\omega}_2}{1-\alpha}\right)\frac{1}{1-\alpha}-f\left(\frac{p-\bar{\omega}_2}{1-\alpha}\right)\frac{1}{1-\alpha}-pf'\left(\frac{p-\bar{\omega}_2}{1-\alpha}\right)\frac{1}{(1-\alpha)^2}\\ &= -2f\left(\frac{p-\bar{\omega}_2}{1-\alpha}\right)\frac{1}{1-\alpha}-pf'\left(\frac{p-\bar{\omega}_2}{1-\alpha}\right)\frac{1}{(1-\alpha)^2}. \end{aligned}\tag{C.51}$$

The expression above is weakly negative if and only if

$$-2f\left(\frac{p-\bar{\omega}_2}{1-\alpha}\right)-pf'\left(\frac{p-\bar{\omega}_2}{1-\alpha}\right)\frac{1}{(1-\alpha)} \leq 0. \tag{C.52}$$

Under a change of variables with $\tilde{p} = \frac{p}{1-\alpha}$ and $\tilde{\omega} = \frac{\bar{\omega}_2}{1-\alpha}$, the previous condition is then equivalent to

$$-2f(\tilde{p}-\tilde{\omega})-\tilde{p}f'(\tilde{p}-\tilde{\omega}) \leq 0. \tag{C.53}$$

This condition is equivalent to Condition (C.50), which holds by assumption. Furthermore, Condition (C.50) additionally implies that Condition (C.53) holds with a strict inequality when $\frac{p-\bar{\omega}_2}{1-\alpha}$ is on the interior of the support of F. This completes the proof of Lemma C.2.

Since F satisfies the assumption of Lemma C.2 (because log-concavity of f implies that F has an increasing hazard rate), M^U is decreasing and thus M_2 is decreasing since it is the convex combination of decreasing functions. This completes Case 1.

Case 2: $p_1^* = \bar{p}$. Suppose the optimal price in period 1 is the price ceiling. Then $p_1^* > p^M$ given that $\bar{p} > p^M$. To show $p_1^* > p_2^*$, suppose that $p_2^* = \bar{p}$ for a contradiction. Recall that if $p_1 = p_2$, then $D^U(p_2;\bar{\omega}_2) = D^I(p_2;\bar{\omega}(s)) \Rightarrow D_2(p_2;\bar{\omega}_2,\bar{\omega}(s)) = D^I(p_2;\bar{\omega}(s))$. Thus, the seller's total profit from $p_1^* = p_2^* = \bar{p}$ would be $2D^I(\bar{p};\bar{\omega}(s)) < 2D^I(p^M;\bar{\omega}(s))$ since p^M uniquely maximizes $pD^I(p;\bar{\omega}(s))$. Thus, $p_1 = p_2 = p^M$ is strictly preferred to $p_1^* = p_2^* = \bar{p}$, contradicting the presumption that the latter path is optimal. Thus, we must have $p_2^* < p_1^*$.

Case 3: p_1^* *interior yet* $p_2^* = p_2^c$. In this case, $p_2^* = p_2^c = (1-\alpha)\underline{v}+\alpha p_1^*$. Note that $p_1^* > p_2^* \Leftrightarrow p_1^* > \underline{v}$, which is true given that is sub-optimal to serve all consumers in period 1. Thus, we need only show that $p_1^* > p^M$ when p_1^* is interior (the ceiling case is considered above). The seller chooses p_1^* to maximize $p_1D^I(p_1;\bar{\omega}(s))+p_2^c[\lambda D^I(p_2;\bar{\omega}(s))+1-\lambda]$, yielding a FOC of

$$M_1(p_1^*;\bar{\omega}(s))+\alpha[\lambda D^I(p_2^c;\bar{\omega}(s))+1-\lambda-\lambda p_2^c f(p_2^c-\bar{\omega}(s))] = 0, \tag{C.54}$$

and thus

$$M_1(p_1^*;\bar{\omega}(s)) + \alpha\lambda M_1(p_2^c;\bar{\omega}(s)) + \alpha[1-\lambda] = 0. \tag{C.55}$$

Recall that $M_1(p^M;\bar{\omega}(s)) = 0$ and $M_1(p;\bar{\omega}(s)) > 0$ for all $p < p^M$. Thus, since $p_2^c < p_1^*$, if $p_1^* \leq p^M$, then $M_1(p_1^*;\bar{\omega}(s)) + \alpha\lambda M_1(p_2^c;\bar{\omega}(s)) > 0$, contradicting the FOC above. This completes the proof of Part 1.

Part 2. Effect of α. First consider the case in which p_1^* and p_2^* are interior solutions to the optimization program in (C.35). From the Envelope Theorem, $\frac{\partial p_n^*}{\partial \alpha} = 0$ for $n = 1, 2$, and hence

$$\frac{\partial}{\partial \alpha}\Pi(p_1, p_2; \alpha, \lambda) = -p_2^* \left[\lambda f(t_2^*)\frac{\partial t^*}{\partial \alpha} + (1-\lambda) f\left(\hat{t}_2\right)\frac{\partial \hat{t}_2}{\partial \alpha}\right], \tag{C.56}$$

where we've defined $t_2^* \equiv p_2 - \bar{\omega}(s)$ and $\hat{t}_2 \equiv \frac{p_2^* - (1-\alpha)\bar{\omega}(s) - \alpha p_1^*}{1-\alpha}$. Since t_2^* is the marginal informed type, $\frac{\partial t^*}{\partial \alpha} = 0$. Now note that

$$\frac{\partial \hat{t}}{\partial \alpha} = \frac{(1-\alpha)[\bar{\omega}(s) - p_1^*] + [p_2^* - (1-\alpha)\bar{\omega}(s) - \alpha p_1^*]}{(1-\alpha)^2} = -\frac{p_1^* - p_2^*}{(1-\alpha)^2}. \tag{C.57}$$

Substituting these values back into (C.56) yields

$$\frac{\partial}{\partial \alpha}\Pi(p_1, p_2; \alpha, \lambda) = (1-\lambda)p_2^* f\left(\frac{p_2^* - (1-\alpha)\bar{\omega}_1 - \alpha p_1^*}{1-\alpha}\right)\left[\frac{p_1^* - p_2^*}{(1-\alpha)^2}\right]. \tag{C.58}$$

Since we have assumed $\lambda < 1$, the expression above is positive whenever $p_1^* > p_2^*$, which is true by Part 1 of this proposition. The case in which $p_1^* = \bar{p}$ and p_2^* is interior yields $\frac{\partial}{\partial \alpha}\Pi(p_1, p_2; \alpha, \lambda)$ that is identical to expression (C.58). Finally, consider the case in which $p_2^* = p_2^c = (1-\alpha)\underline{v} + \alpha p_1$ (i.e., the corner case described in Part 1 in which all uninformed types are served in period 2). In period 1, the seller chooses p_1 to maximize

$$\Pi^c(p_1; \alpha, \lambda) = p_1[1 - F(p_1 - \bar{\omega}(s))] + [(1-\alpha)\underline{v} + \alpha p_1]\Big[1 - \lambda F((1-\alpha)\underline{v} + \alpha p_1 - \bar{\omega}(s))\Big]. \tag{C.59}$$

Note that this profit function accounts for the fact that all uninformed types buy in period 2. Let p_1^* be the value of p_1 that maximizes the expression above, and let $p_2^c(p_1^*) \equiv (1-\alpha)\underline{v} + \alpha p_1^*$. For either an interior value p_1^* or $p_1^* = \bar{p}$, we have

$$\frac{\partial \Pi^c(p_1; \alpha, \lambda)}{\partial \alpha} = (p_1^* - \underline{v})\Big[1 - \lambda F(p_2^c(p_1^*) - \bar{\omega}(s))\Big] - p_2^c(p_1^*)\lambda f\Big(p_2^c(p_1) - \bar{\omega}(s)\Big)(p_1^* - \underline{v}), \tag{C.60}$$

and hence $\frac{\partial}{\partial \alpha}\Pi^c(p_1; \alpha, \lambda) > 0$ if and only if

$$\Big[1 - \lambda F(p_2^c(p_1^*) - \bar{\omega}(s))\Big] - p_2^c(p_1^*)\lambda f\Big(p_2^c(p_1^*) - \bar{\omega}(s)\Big) > 0. \tag{C.61}$$

The previous condition must hold given that we are focusing on the case in which all uninformed types are served: as argued above, it is optimal to set the highest possible price in the first region of D_2 in (C.38), and hence the previous inequality must hold for all $p_2 \leq (1-\alpha)\underline{v} + \alpha p_1$.

Effect of λ. Similar to the approach above, if p_2^* is interior and either p_1^* is interior or $p_1^* = \bar{p}$, then we have

$$\frac{\partial}{\partial \lambda}\Pi(p_1, p_2; \alpha, \lambda) = p_2^* \left[-F(t_2^*) + F\left(\hat{t}_2\right)\right], \tag{C.62}$$

where neither t_2^* nor $\hat{t}_2$ depend on λ. This expression is negative whenever $\hat{t}_2 < t_2^*$. Notice that

$$\hat{t}_2 = \frac{p_2^* - (1-\alpha)\bar{\omega}(s) - \alpha p_1^*}{1-\alpha} = p_2^* - \bar{\omega}(s) - \frac{\alpha}{1-\alpha}[p_1^* - p_2^*] = t_2^* - \frac{\alpha}{1-\alpha}[p_1^* - p_2^*]. \tag{C.63}$$

Since $\alpha > 0$, $\hat{t}_2 < t_2^* \Leftrightarrow p_1^* - p_2^* > 0$, which is again true by Part 1 of this proposition. If instead we have a corner solution in period 2, then the profit function is as in (C.59) and

$$\frac{\partial \Pi^c(p_1; \alpha, \lambda)}{\partial \lambda} = -p_2^c(p_1^*) F\bigg(p_2^c(p_1) - \bar{\omega}(s)\bigg), \tag{C.64}$$

which is clearly negative. ■

Proof of Proposition 6. *Part 1.* Consider the optimal price pair (p_1^*, p_2^*). Let $t_2^* \equiv p_2^* - \bar{\omega}(s)$ denote the marginal informed type in period 2, and and let $\hat{t}_2 \equiv \frac{p_2^* - \bar{\omega}_2}{1-\alpha}$ denote the marginal uninformed type. Note that if $\hat{t}_2 < t_2^*$, then the interval of types who adopt the good in period 2 at a price above their true expected valuation is $[\hat{t}_2, t_2^*]$. From (C.63), we have $t_2^* - \hat{t}_2 = \frac{\alpha}{1-\alpha}[p_1^* - p_2^*]$. Since $p_1^* - p_2^* > 0$ for all $\alpha > 0$ (by Proposition 5 Part 1), we know that $\hat{t}_2 < t_2^*$. Thus, the width of the interval of types who wrongly adopt is

$$t_2^* - \hat{t}_2 = \frac{\alpha}{1-\alpha}[p_1^* - p_2^*], \tag{C.65}$$

which is strictly positive.

Part 2. Suppose that $\bar{\omega}(s) + \underline{t} < 0$. We show that an α sufficiently large will induce the seller to set the "corner" price in period 2 at which all uninformed types are served. Recall from the proof of Proposition 5 that this price is $p_2^c = (1-\alpha)\underline{v} + \alpha p_1$, where $\underline{v} = \bar{\omega}(s) + \underline{t}$. We will show that the price derivative of the period-2 profit function is necessarily negative at p_2^c for α sufficiently large, implying that $p_2^* = p_2^c$, and thus that all uninformed types are served. Toward this end, recall that the period-2 profit is

$$\Pi_2(p_2; p_1) = p_2\bigg(1 - \lambda F(p_2 - \bar{\omega}(s)) - (1-\lambda) F\left(\frac{p_2 - (1-\alpha)\bar{\omega}(s) - \alpha p_1}{1-\alpha}\right)\bigg), \tag{C.66}$$

and hence

$$\begin{aligned}\frac{\partial \Pi_2(p_2; p_1)}{\partial p_2} = &\left(1 - \lambda F(p_2 - \bar{\omega}(s)) - (1-\lambda) F\left(\frac{p_2 - (1-\alpha)\bar{\omega}(s) - \alpha p_1}{1-\alpha}\right)\right) \\ &- p_2\bigg(\lambda f(p_2 - \bar{\omega}(s)) + \frac{(1-\lambda)}{(1-\alpha)} f\left(\frac{p_2 - (1-\alpha)\bar{\omega}(s) - \alpha p_1}{1-\alpha}\right)\bigg).\end{aligned} \tag{C.67}$$

To evaluate $\frac{\partial \Pi_2(p_2;p_1)}{\partial p_2}\Big|_{p_2=p_2^c}$, notice that $\frac{p_2^c-(1-\alpha)\bar{\omega}(s)-\alpha p_1}{1-\alpha}=\underline{t}$. Since $F(\underline{t})=0$, we have

$$\frac{\partial \Pi_2(p_2;p_1)}{\partial p_2}\bigg|_{p_2=p_2^c} = 1-\lambda\bigg(F(p_2^c-\bar{\omega}(s))-p_2^c f(p_2^c-\bar{\omega}(s))\bigg)-p_c^2\frac{(1-\lambda)}{(1-\alpha)}f\left(\underline{t}\right), \tag{C.68}$$

and thus a sufficient condition for $\frac{\partial \Pi_2(p_2;p_1)}{\partial p_2}\Big|_{p_2=p_2^c}<0$ is

$$p_c^2\frac{(1-\lambda)}{(1-\alpha)}f\left(\underline{t}\right)>1. \tag{C.69}$$

Since $p_2^c=(1-\alpha)\underline{v}+\alpha p_1$, the previous condition is equivalent to

$$\underline{v}+\frac{\alpha}{(1-\alpha)}p_1>\frac{1}{(1-\lambda)f(\underline{t})}. \tag{C.70}$$

From Proposition 5 Part 1, we know that along the optimal price path, $p_1>p^M$ for all $\alpha>0$. Hence, a sufficient condition for (C.70) is

$$\underline{v}+\frac{\alpha}{(1-\alpha)}p^M>\frac{1}{(1-\lambda)f(\underline{t})}. \tag{C.71}$$

The right-hand side of (C.71) is positive and finite given that f is positive on $\mathcal{T}$. Thus, since $p^M>0$, there exists $\tilde{\alpha}\in(0,1)$ such that $\underline{v}+\frac{\tilde{\alpha}}{(1-\tilde{\alpha})}p^M=\frac{1}{(1-\lambda)f(\underline{t})}$. Then $\alpha>\tilde{\alpha}$ implies that Condition (C.71) holds, and hence the seller chooses p_2^c such that all uninformed types are served in period 2. ■

Proof of Lemma 2. As noted in the text, we restrict attention to the case in which it is never optimal to serve the lowest type. In this case, Equation (13) implies that the true demand function in period $n\geq 2$ is $D(p_n;\bar{\omega}_n,\bar{\omega}(s))=\lambda D^I(p_n;\bar{\omega}(s))+(1-\lambda)D^U(p_n;\bar{\omega}_n)$, where D^I and D^U are specified in Equation (16). Hence,

$$D(p_n;\bar{\omega}_n,\bar{\omega}(s))=\frac{(1-\alpha)\bar{t}+\lambda(1-\alpha)\bar{\omega}(s)+(1-\lambda)\bar{\omega}_n-(1-\lambda\alpha)p_n}{(1-\alpha)(\bar{t}-\underline{t})}. \tag{C.72}$$

In period $n+1$, an uninformed observer with taste t thinks that when the preceding generation holds a common expectation of ω equal to $\hat{\omega}$, then their demand is given by

$$\widehat{D}(p_n;\hat{\omega}|t)=\frac{(1-\alpha)\bar{t}+\hat{\omega}-p_n+\alpha t}{(1-\alpha)(\bar{t}-\underline{t})}. \tag{C.73}$$

The inferred value of this observer, denoted $\hat{\omega}_{n+1}(t)$, is the value of $\hat{\omega}$ that solves $\widehat{D}(p_n;\hat{\omega}|t)=D(p_n;\bar{\omega}_n,\bar{\omega}(s))$. By Lemma 1, $\hat{\omega}_{n+1}(t)=\bar{\omega}_{n+1}-\alpha t$. Substituting this into the previous equality and solving for $\bar{\omega}_{n+1}$ in terms of $\bar{\omega}_n$ yields the following law of motion:

$$\bar{\omega}_{n+1}=\lambda\big[(1-\alpha)\bar{\omega}(s)+\alpha p_n\big]+(1-\lambda)\bar{\omega}_n, \tag{C.74}$$

starting from $\bar{\omega}_2 = (1-\alpha)\bar{\omega}(s) + \alpha p_1$. We complete the proof using induction on $n \geq 2$. Define

$$\tilde{p}^{n-1} \equiv (1-\lambda)^{n-2} p_1 + \sum_{k=2}^{n-1} \lambda(1-\lambda)^{n-1-k} p_k. \tag{C.75}$$

For the base case, note that (C.74) implies that $\bar{\omega}_3 = \lambda\big[(1-\alpha)\bar{\omega}(s) + \alpha p_2\big] + (1-\lambda)[(1-\alpha)\bar{\omega}(s) + \alpha p_1] = (1-\alpha)\bar{\omega}(s) + \alpha[(1-\lambda)p_1 + \lambda p_2] = (1-\alpha)\bar{\omega}(s) + \alpha\tilde{p}^2$. Now suppose that for any $n > 2$, we have $\bar{\omega}_n = (1-\alpha)\bar{\omega}(s) + \alpha\tilde{p}^{n-1}$. Again, (C.74) implies that $\bar{\omega}_{n+1} = \lambda\big[(1-\alpha)\bar{\omega}(s) + \alpha p_n\big] + (1-\lambda)[(1-\alpha)\bar{\omega}(s) + \alpha\tilde{p}^{n-1}] = (1-\alpha)\bar{\omega}(s) + \alpha[(1-\lambda)\tilde{p}^{n-1} + \lambda p_n] = (1-\alpha)\bar{\omega}(s) + \alpha\tilde{p}^n$. ■

Proof of Proposition 7. As noted in the text, we restrict attention to the case in which it is never optimal to serve the lowest type. Thus, the optimal price path is characterized by the first-order conditions, aside from the possibility of pricing at the ceiling. We discuss the price-ceiling case at the end of the proof and focus on the interior case first. In the interior case, profit in period $n \geq 2$ is

$$\Pi(p_n; \bar{\omega}_n, \bar{\omega}(s)) = p_n D(p_n; \bar{\omega}_n, \bar{\omega}(s)) = p_n \left(\frac{(1-\alpha)\bar{t} + \lambda(1-\alpha)\bar{\omega}(s) + (1-\lambda)\bar{\omega}_n - (1-\lambda\alpha)p_n}{(1-\alpha)(\bar{t} - \underline{t})} \right);$$

in period $n = 1$, profit is $\widetilde{\Pi}(p_1; \bar{\omega}(s)) = p_1 \left(\frac{\bar{t} + \bar{\omega}_1 - p_1}{\bar{t} - \underline{t}} \right)$. The seller's maximization problem is thus

$$\max_{\{p_n\}_{n=1}^N} \left(\widetilde{\Pi}(p_1; \bar{\omega}(s)) + \sum_{n=2}^N \Pi(p_n; \bar{\omega}_n) \right) \quad \text{s.t.} \quad \bar{\omega}_{n+1} = \varphi(\bar{\omega}_n, p_n)\ \forall n = 2, \ldots, N, \tag{C.76}$$

where $\varphi(\bar{\omega}_n; p_n) \equiv \lambda\big[(1-\alpha)\bar{\omega}(s) + \alpha p_n\big] + (1-\lambda)\bar{\omega}_n$ is the transition function derived in Lemma 2. The Lagrangian is then

$$\mathcal{L} = \widetilde{\Pi}(p_1; \bar{\omega}_1) + \sum_{n=2}^N \Pi(p_n; \bar{\omega}_n) + \sum_{n=1}^N \gamma_n(\bar{\omega}_{n+1} - \varphi(\bar{\omega}_n, p_n)), \tag{C.77}$$

where $\{\gamma_n\}_{n=1}^N$ are Lagrange multipliers.

The plan for the proof is to first develop a set of equations (first-order conditions and Euler equations) that characterize the optimal price path. We will then argue that the price in the final period, p_N, must be lower than p_{N-1} by the same logic underlying the two-period case (Proposition 5). We then argue by induction that if for any n we have $p_n > p_{n+1} > \cdots > p_N$, then $p_{n-1} > p_n$, which establishes the declining price path (i.e. Part 2 of the proposition). Finally, we will note that $p_1 > p_M$ (i.e,. Part 1).

We begin by deriving a set of first-order conditions that characterize the system of prices. Given the functional forms of Π, $\widetilde{\Pi}$, and φ, we have the following collection of first-order conditions: (i) the FOC w.r.t. p_1 is

$$\frac{\bar{t} + \bar{\omega}(s) - 2p_1}{\bar{t} - \underline{t}} = \gamma_1 \alpha; \tag{C.78}$$

(ii) the FOC w.r.t. p_n for $n = 2, \ldots, N-1$ is

$$\left(\frac{(1-\alpha)\bar{t} + \lambda(1-\alpha)\bar{\omega}(s) + (1-\lambda)\bar{\omega}_n - 2(1-\lambda\alpha)p_n}{(1-\alpha)(\bar{t} - \underline{t})} \right) = \gamma_n \lambda\alpha; \tag{C.79}$$

(iii) the FOC w.r.t. p_N is

$$\left(\frac{(1-\alpha)\bar{t}+\lambda(1-\alpha)\bar{\omega}(s)+(1-\lambda)\bar{\omega}_N-2(1-\lambda\alpha)p_N}{(1-\alpha)(\bar{t}-\underline{t})}\right)=0, \tag{C.80}$$

which follows from the fact that $\gamma_N = 0$ given the FOC w.r.t. to $\bar{\omega}_{N+1}$; and (iv) the FOC w.r.t. $\bar{\omega}_n$ for $n = 2, \ldots, N$ is

$$p_n\left(\frac{1-\lambda}{(1-\alpha)(\bar{t}-\underline{t})}\right)+\gamma_{n-1}=\gamma_n(1-\lambda). \tag{C.81}$$

From these FOCs, we can derive an "Euler equation" by using the FOC for p_{n-1} in (C.79) to solve for γ_{n-1} and then substituting this value into (C.81). The result provides a link between p_{n-1} and p_n in terms of the current beliefs. Equations (C.78) and (C.81)—along with the fact that $p^M = (\bar{t} + \bar{\omega}(s))/2$—imply that the Euler equation linking periods 1 and 2 is

$$p_2=\left(\frac{2\lambda(1-\alpha)+\alpha(1-\lambda)^2}{(1-\lambda)(2-\lambda\alpha)}\right)p_1-\frac{2(2\lambda-1)(1-\alpha)}{(1-\lambda)(2-\lambda\alpha)}p^M. \tag{C.82}$$

For $n > 2$, equations (C.79) and (C.81) along with the expression for $\bar{\omega}_n$ in terms of past prices (from Lemma 2) imply that the Euler equation linking periods $n-1$ and n is:

$$p_n=\phi_{-1}p_{n-1}-\phi_M p^M-\tilde{\phi}\tilde{p}^{n-2} \tag{C.83}$$

where we've introduced the following positive constants:

$$\begin{aligned}
\phi_{-1} &= \frac{(2-\alpha\lambda)-\alpha\lambda^2(2-\lambda)}{(1-\lambda)(2-\alpha\lambda)}, && \text{(C.84)}\\
\phi_M &= \frac{2\lambda(1-\alpha)}{(1-\lambda)(2-\lambda\alpha)}, && \text{(C.85)}\\
\tilde{\phi} &= \alpha\frac{\lambda(2-\lambda)}{(2-\lambda\alpha)}. && \text{(C.86)}
\end{aligned}$$

To characterize the solution, we will combine these Euler equations with the FOCs for each p_n. Using our expression for $\bar{\omega}_n$ in terms of past prices (from Lemma 2), the FOCs w.r.t. p_n for $n \geq 2$ from above can be equivalently written as

$$\begin{aligned}
0 &= (1-\alpha)(\bar{t}+\bar{\omega}(s))+\alpha(1-\lambda)\tilde{p}^{n-1}-2(1-\lambda\alpha)p_n+\alpha(1-\lambda)\sum_{k=n+1}^{N}p_k\frac{\partial\tilde{p}^{k-1}}{\partial p_n}\\
&= 2(1-\alpha)p^M+\alpha(1-\lambda)\tilde{p}^{n-1}-2(1-\lambda\alpha)p_n+\alpha\lambda\sum_{k=n+1}^{N}(1-\lambda)^{k-n}p_k, && \text{(C.87)}
\end{aligned}$$

where we've used the fact that $\frac{\partial\tilde{p}^{k-1}}{\partial p_n}=\lambda(1-\lambda)^{k-n-1}$ and $p^M = (\bar{t} + \bar{\omega}(s))/2$ in the uniform case.

Given that the demand function in period 1 is different from the one in $n \geq 2$, the FOC w.r.t. p_1 is

$$0 = (1-\alpha)p^M - 2(1-\alpha)p_1 + \alpha \sum_{k=2}^{N} (1-\lambda)^{k-1} p_k \tag{C.88}$$

since $\frac{\partial \tilde{p}^{k-1}}{\partial p_1} = (1-\lambda)^{k-2}$. To summarize, the N prices must solve the following system of N equations:

$$\begin{aligned}
p_1 &= p^M + \frac{\alpha}{2(1-\alpha)} \left(\sum_{k=2}^{N} (1-\lambda)^{k-1} p_k \right) \\
&\vdots \\
p_n &= \left(\frac{1-\alpha}{1-\lambda\alpha} \right) p^M + \left(\frac{\alpha}{2(1-\lambda\alpha)} \right) \left((1-\lambda)\tilde{p}^{n-1} + \lambda \sum_{k=n+1}^{N} (1-\lambda)^{k-n} p_k \right) \\
&\vdots \\
p_N &= \left(\frac{1-\alpha}{1-\lambda\alpha} \right) p^M + \left(\frac{\alpha}{2(1-\lambda\alpha)} \right) \left((1-\lambda)\tilde{p}^{N-1} \right).
\end{aligned} \tag{C.89}$$

Going forward, we will streamline notation by letting $c_n \equiv p_n/p^M$ denote the "normalized" price in each period n. This allows us to characterize the system for $(c_1, \ldots, c_N)$ without any explicit dependence on the value of p^M. Similarly, for all n, let $\tilde{c}^{n-1} = \tilde{p}^{n-1}/p^M = (1-\lambda)^{n-2}c_1 + \sum_{k=2}^{n-1} \lambda(1-\lambda)^{n-1-k} c_k$. Additionally, let $\hat{c}^{n+1} \equiv \sum_{k=n+1}^{N} (1-\lambda)^{k-n} p_k/p^M = \sum_{k=n+1}^{N} (1-\lambda)^{k-n} c_k$.

We now prove the following via induction: for $n > 2$, if $c_n > c_{n+1} > \cdots > c_N$, then $c_{n-1} > c_n$.

Base Case: $c_{N-1} > c_N$. We prove the base case by showing $c_{N-1} > c_N$. From (C.87), the FOC w.r.t. c_{N-1} is $2(1-\alpha) + \alpha(1-\lambda)\tilde{c}^{N-2} - 2(1-\lambda\alpha)c_{N-1} + \alpha\lambda(1-\lambda)c_N = 0$, and the FOC w.r.t. c_N is $2(1-\alpha) + \alpha(1-\lambda)\tilde{c}^{N-1} - 2(1-\lambda\alpha)c_N = 0$. The definition of $\tilde{c}^{N-1}$ implies that that $\tilde{c}^{N-1} = (1-\lambda)\tilde{c}^{N-2} + \lambda c_{N-1}$. Substituting this value into the latter FOC and equating the two FOCs yields the following necessary condition:

$$\alpha\lambda(1-\lambda)\tilde{c}^{N-2} = \Big(2(1-\lambda\alpha) + \alpha\lambda(1-\lambda) \Big) [c_{N-1} - c_N]. \tag{C.90}$$

It is straightforward to verify that $2(1-\lambda\alpha) + \alpha\lambda(1-\lambda) = 2 - \alpha\lambda[1+\lambda] > 0$ for any $\alpha \in (0,1)$ and any $\lambda \in (0,1)$. Thus, since the left-hand side of (C.90) is strictly positive (it is a weighted sum of normalized prices), we have $c_{N-1} > c_N$.

Induction step: $c_n > c_{n+1}$ *for* $n \geq 2$. Consider $n \in \{3, \ldots, N-1\}$ and suppose that $c_n > c_{n+1} > \cdots > c_N$. We will show that $c_{n-1} > c_n$. To do so, we first derive an expression for c_{n-1} purely in terms of $(c_n, \ldots, c_N)$. Note that neither the Euler equation for c_{n-1} nor the FOC w.r.t. c_{n-1} provides this: the former characterizes c_{n-1} as a function of previous prices, $(c_1, \ldots, c_{n-1})$ and the latter characterizes c_{n-1} as a function of previous *and* future prices. To obtain this expression, note that (C.83) implies $\tilde{c}^{n-2} = (\phi_{-1}c_{n-1} - c_n - \phi_M)/\tilde{\phi}$. Substituting this value into the FOC w.r.t.

c_{n-1} (Equation C.87) yields

$$2(1-\lambda\alpha)c_{n-1} = 2(1-\alpha) + \alpha(1-\lambda)\frac{1}{\tilde{\phi}}\bigg(\phi_{-1}c_{n-1} - c_n - \phi_M\bigg) + \alpha\lambda\hat{c}^n. \tag{C.91}$$

From the definition of $\hat{c}^n$, note that $\hat{c}^n = (1-\lambda)c_n + (1-\lambda)\hat{c}^{n+1}$. Substituting this expression into (C.91) and substituting the values of the constants ϕ_{-1}, ϕ_M, and $\tilde{\phi}$ from above (Equations C.84 to C.86), and simplifying, reveals that

$$c_{n-1} = \phi_{-1}c_n + \phi_M - \left(\frac{\lambda}{1-\lambda}\right)\tilde{\phi}\hat{c}^{n+1}. \tag{C.92}$$

Recall that, by assumption, $c_n > c_{n+1} > \cdots > c_N$, and we want to show $c_{n-1} > c_n$. From (C.92), this condition is equivalent to $\phi_{-1}c_n + \phi_M - \left(\frac{\lambda}{1-\lambda}\right)\tilde{\phi}\hat{c}^{n+1} > c_n$, and hence equivalent to

$$[\phi_{-1} - 1]c_n > \left(\frac{\lambda}{1-\lambda}\right)\tilde{\phi}\hat{c}^{n+1} - \phi_M. \tag{C.93}$$

From the definition of ϕ_{-1}, we have $\phi_{-1} - 1 > 0$. Notice that (C.92) must hold for all $n \in \{3, \ldots, N-1\}$, and hence $c_n = \phi_{-1}c_{n+1} + \phi_M - \left(\frac{\lambda}{1-\lambda}\right)\tilde{\phi}\hat{c}^{n+2}$. Moreover, note that the definitions of ϕ_{-1} and $\tilde{\phi}$ are such that $\phi_{-1} = (1-\lambda\tilde{\phi})/(1-\lambda)$; substituting this into the previous equality along with the fact that $\hat{c}^{n+1} = (1-\lambda)c_{n+1} + (1-\lambda)\hat{c}^{n+1}$ implies that $\left(\frac{\lambda}{1-\lambda}\right)\tilde{\phi}\hat{c}^{n+1} = -(1-\lambda)c_n + (1-\lambda)\phi_M + c_{n+1}$. Substituting this into the inequality of interest (Condition C.93) yields the equivalent condition of $[\phi_{-1} - \lambda]c_n > c_{n+1} - \lambda\phi_M$. Since we know $c_n > c_{n+1}$ and since $\phi_{-1} - \lambda > 0$ (because $\phi_{-1} > 1$, as noted above), the previous condition will hold at $c_n > c_{n+1}$ if it holds at $c_n = c_{n+1}$. Thus, it suffices to show that $[\phi_{-1} - \lambda]c_{n+1} > c_{n+1} - \lambda\phi_M \Leftrightarrow [\phi_{-1} - \lambda - 1]c_{n+1} > -\lambda\phi_M$. The previous condition holds so long as $\phi_{-1} - \lambda - 1 > 0$, which can be directly confirmed from the definition of ϕ_{-1} in (C.84). This completes the induction step.

So far, we have verified that $c_{N-1} > c_N$ implies that $c_n > c_{n+1}$ for all $n \geq 2$. To complete the proof, we must show that $c_2 > c_3 > \cdots > c_N$ implies that $c_1 > c_2$. Since the Euler equation linking periods 1 and 2 is different from one in all other periods, we cannot rely on (C.92) as above. Instead, consider the FOCs in periods 1 and 2 (Equations C.88 and C.87), which are $2(1-\alpha) - 2(1-\alpha)c_1 + \alpha\hat{c}^2 = 0$ and $2(1-\alpha) + \alpha(1-\lambda)\tilde{c}^1 - 2(1-\lambda\alpha)c_2 + \alpha\lambda\hat{c}^3 = 0$, respectively. Using the fact that $\hat{c}^2 = (1-\lambda)c_2 + (1-\lambda)\hat{c}^3$, equating the two FOCs and simplifying yields the condition

$$\alpha[(1-2\lambda)\hat{c}^2 + 2(1-\lambda)c_2] = \zeta[c_1 - c_2], \tag{C.94}$$

where $\zeta = [2(1-\alpha) + \alpha(1-\lambda)] = 2 - \alpha(1+\lambda)$; note that $\zeta \in (0,2)$ for all $\alpha \in (0,1)$. Thus, we have $c_1 > c_2$ so long as $(1-2\lambda)\hat{c}^2 + 2(1-\lambda)c_2 > 0 \Leftrightarrow 2(1-\lambda)c_2 > (2\lambda-1)\hat{c}^3$. While this holds immediately whenever $\lambda < 1/2$, we must show it holds more generally. Recall that $\hat{c}^3 = \sum_{k=3}^{N}(1-\lambda)^{k-2}c_k$. Substituting this into the previous inequality yields the equivalent condition of $2(1-\lambda)c_2 > (2\lambda-1)\sum_{k=3}^{N}(1-\lambda)^{k-2}c_k \Leftrightarrow 2c_2 > (2\lambda-1)\sum_{k=3}^{N}(1-\lambda)^{k-3}c_k$. Since we've assumed $c_2 > c_3 > \cdots > c_N$, a sufficient condition for the previous inequality is

$$2c_2 > (2\lambda-1)c_2\sum_{k=3}^{N}(1-\lambda)^{k-3} \Leftrightarrow 2 > (2\lambda-1)\sum_{k=0}^{N-3}(1-\lambda)^k. \tag{C.95}$$

Recall that the partial sum of the geometric series is $\sum_{k=0}^{N-3}(1-\lambda)^k$ is strictly less than $\frac{1}{1-(1-\lambda)} = \frac{1}{\lambda}$. Thus, a sufficient condition for Condition (C.95) is $2 > (2\lambda - 1)\frac{1}{\lambda}$, which necessarily holds.

Finally, it is immediate from the FOC for p_1 in (C.87) that $p_1 > p^M$. Similarly, if the FOC in period 1 does not hold because the seller prefers setting p_1 equal to the price ceiling, $\bar{p}$, then the logic of this proof remains unchanged. If $p_1 = \bar{p}$, then clearly we have $p_1 > p^M$; moreover, the seller would never charge $p_2 = \bar{p}$ if $p_1 = \bar{p}$ since she strictly profits from a price decrease in period 2. Thus, it is immediate that we still have $p_2 < p_1 = \bar{p}$ in this case, and hence prices will follow the interior path described above from period 2 onward. ■

Proof of Proposition 8. In period 1, the quantity demanded is

$$D_1(p;\bar{\omega}(s)) = \lambda[1 - F(p - \bar{\omega}(s))] + (1-\lambda)[1 - F(p - \bar{\omega}_0)]. \tag{C.96}$$

Now consider what an agent with taste t who delays will infer from observing this quantity. They think that if informed agents expect a quality of $\hat{\omega}$; then the demand in period 1 is

$$\widehat{D}_1(p;\hat{\omega}|t) = \lambda\left[1 - F\left(\frac{p - \hat{\omega} - \alpha t}{1-\alpha}\right)\right] + (1-\lambda)\left[1 - F\left(\frac{p - \bar{\omega}_0 - \alpha t}{1-\alpha}\right)\right]. \tag{C.97}$$

Equating the two equations above allows us to solve for $\hat{\omega}_2(t)$, which denotes the perceived quality of an agent with taste t who has not bought in period 1. Assuming $T \sim U(\underline{t}, \bar{t})$, this solution is

$$\hat{\omega}_2(t) = \frac{\alpha}{\lambda}\bigg(p - (1-\lambda)\bar{\omega}_0 - t\bigg) + (1-\alpha)\bar{\omega}(s). \tag{C.98}$$

The marginal type in period 2 under projection is the $\hat{t}_2$ that solves $\hat{\omega}_2(\hat{t}_2) + \hat{t}_2 = p$, and hence

$$\hat{t}_2 = p - \left[\frac{\lambda(1-\alpha)}{\lambda-\alpha}\right]\bar{\omega}(s) + \left[\frac{\alpha(1-\lambda)}{\lambda-\alpha}\right]\bar{\omega}_0. \tag{C.99}$$

The marginal type in period 2 under rational inference is $t_2^* = p - \bar{\omega}(s)$. Note that $\hat{t}_2 < t_2^*$ if and only if

$$p - \left[\frac{\lambda(1-\alpha)}{\lambda-\alpha}\right]\bar{\omega}(s) + \left[\frac{\alpha(1-\lambda)}{\lambda-\alpha}\right]\bar{\omega}_0 < p - \bar{\omega}(s) \quad\Leftrightarrow\quad \bar{\omega}(s) > \bar{\omega}_0. \tag{C.100}$$

Recall that the only types present in period 2 are those who did not buy in period 1; i.e., only those with $t \leq t_1^U \equiv p - \bar{\omega}_0$. Note that rational consumers in period 2 buy if and only if $t_2^* < t_2^U \Leftrightarrow \bar{\omega}(s) > \bar{\omega}_0$. Condition (C.100) thus implies that the same is true under projection: $\hat{t}_2 < t_2^U \Leftrightarrow \bar{\omega}(s) > \bar{\omega}_0$; hence, projectors in period 2 only buy when the quality is higher than expected.

Part 1. Suppose $\bar{\omega}(s) > \bar{\omega}_0$. Under rational inference, the interval of types who buy in period 2 is $[t_2^*, t_1^U]$. Under projection, this interval is $[\hat{t}_2, t_1^U]$, where $\hat{t}_2 < t_2^*$ by (C.100). Hence, the quantity demanded in period 2 under projection exceeds the rational benchmark. Moreover, using the expressions above for $\hat{t}_2$ and t_2^*, the interval of types who wrongly adopt the good is

$$t_2^* - \hat{t}_2 = \frac{\alpha(1-\lambda)}{\lambda-\alpha}\big[\bar{\omega}(s) - \bar{\omega}_0\big]. \tag{C.101}$$

The measure of this interval is clearly increasing in α and in $\bar{\omega}(s) - \bar{\omega}_0$.

Now consider the range of types who buy in period 2 yet hold a quality expectation that exceeds the rational expectation, $\mathcal{T}_O \equiv \{t \in [\hat{t}, t_1^U] \mid \tilde{\omega}_2(t) > \bar{\omega}(s)\}$. This set represents the buyers who overestimate quality and will, on average, be disappointed by adoption ex post; that is, $t \in \mathcal{T}_O \Rightarrow \mathbb{E}[\omega - \hat{\omega}(t)|s] < 0$. Let $\tilde{t}$ be the type in period 2 who infers correctly; i.e., $\hat{\omega}_2(\tilde{t}) = \bar{\omega}(s)$. From (C.98), we have

$$\tilde{t} = p - \lambda\bar{\omega}(s) - (1-\lambda)\bar{\omega}_0. \tag{C.102}$$

Since $\hat{\omega}_2(t)$ is decreasing in t, all types $t < \tilde{t}$ in period 2 will overestimate quality and hence $\mathcal{T}_O = [\hat{t}, \tilde{t})$. Since $\bar{\omega}(s) > \bar{\omega}_0$, we have $\tilde{t} \in (t_2^*, t_1^U)$ given that $\lambda \in (0,1)$. In contrast to rational learning, $\tilde{t} > t_2^*$ implies that some projecting buyers who correctly adopt the good (i.e., their expected valuation exceeds the price) will systematically experience disappointment, on average.

Part 2. Suppose $\bar{\omega}(s) < \bar{\omega}_0$. As discussed prior to Part 1, $\bar{\omega}(s) < \bar{\omega}_0$ implies that no consumers buy in period 2 under rational inference or under projection. Hence, outcomes in this case match the rational benchmark. ■

Proof of Proposition 9. Before proving the proposition, we derive some preliminary results on the nature of uninformed agents' biased inference rules and the equilibrium quantity demanded.

Let $t^* \equiv p - \bar{\omega}(s)$ be the marginal informed type (i.e., an informed type strictly prefers to buy a positive quantity if and only if $t > t^*$). The aggregate demand of informed agents is then

$$\begin{aligned} D^I(p;\bar{\omega}(s)) &= \int_{\mathcal{T}} x^*(p;\bar{\omega}(s),t)dF(t) = \int_{t^*}^{\bar{t}} (\bar{\omega}(s) - p + t)dF(t) \\ &= -[1-F(t^*)]t^* + \int_{t^*}^{\bar{t}} \tilde{t} f(\tilde{t}) d\tilde{t}. \end{aligned} \tag{C.103}$$

Let $H(t) \equiv -[1-F(t)]t + \int_{\tilde{t}\geq t} \tilde{t} f(d) d\tilde{t}$. Now consider the demand function among agents with a quality expectation of $\hat{\omega}$ from the perspective of an uninformed agent with taste t. This agent believes the marginal type is $\hat{t} = p - \hat{\omega}$, and hence perceives

$$\begin{aligned} \widehat{D}^I(p;\hat{\omega}|t) &= -[1-\widehat{F}(\hat{t}|t)]\hat{t} + \int_{\hat{t}}^{\bar{t}(t)} \tilde{t}\hat{f}(\tilde{t}|t)d\tilde{t} \\ &= -\left[1 - F\left(\frac{\hat{t}-\alpha t}{1-\alpha}\right)\right]\hat{t} + \int_{\hat{t}}^{\bar{t}(t)} \tilde{t}\frac{1}{1-\alpha} f\left(\frac{\tilde{t}-\alpha t}{1-\alpha}\right) d\tilde{t}. \end{aligned} \tag{C.104}$$

Consider a change of variables with $x = \frac{\tilde{t}-\alpha t}{1-\alpha}$. Recalling that $\bar{t}(t) = \alpha t + (1-\alpha)\bar{t}$, expression

(C.104) can be re-written as

$$
\begin{aligned}
\widehat{D}^I(p;\hat{\omega}|t) &= -\left[1-F\left(\frac{\hat{t}-\alpha t}{1-\alpha}\right)\right]\hat{t}+\int_{\frac{\hat{t}-\alpha t}{1-\alpha}}^{\bar{t}}[\alpha t+(1-\alpha)x]f(x)\,dx \\
&= -\left[1-F\left(\frac{\hat{t}-\alpha t}{1-\alpha}\right)\right][\hat{t}-\alpha t]+(1-\alpha)\int_{\frac{\hat{t}-\alpha t}{1-\alpha}}^{\bar{t}}xf(x)\,dx \\
&= (1-\alpha)\Bigg(-\left[1-F\left(\frac{\hat{t}-\alpha t}{1-\alpha}\right)\right]\left(\frac{\hat{t}-\alpha t}{1-\alpha}\right)+\int_{\frac{\hat{t}-\alpha t}{1-\alpha}}^{\bar{t}}xf(x)\,dx\Bigg) \\
&= (1-\alpha)H\left(\frac{\hat{t}-\alpha t}{1-\alpha}\right), \qquad \text{(C.105)}
\end{aligned}
$$

where H is defined in (C.103).

An uninformed projecting agent's inference rule, $\hat{\omega}(d|t)$, is obtained by solving for the perceived marginal type $\hat{t}(d|t)$ that solves $\widehat{D}^I(p;\hat{\omega}|t)=(1-\alpha)H\left(\frac{\hat{t}-\alpha t}{1-\alpha}\right)=d$, and then setting $\hat{\omega}(d|t)=p-\hat{t}$. We now use the Implicit Function Theorem (IFT) to show that a projector's biased inference rule is linearly decreasing in t with slope α.

Let $L(x;d)=(1-\alpha)H(x)-d$. Note that an agent infers a marginal type $\hat{t}(d|t)$ equal to the value of $\hat{t}$ that solves $L\left(\frac{\hat{t}-\alpha t}{1-\alpha};d\right)=0$. Thus,

$$
\begin{aligned}
\frac{\partial\hat{t}(d|t)}{\partial t} &= -\left(\frac{\partial}{\partial t}L\left(\frac{\hat{t}-\alpha t}{1-\alpha};d\right)\right)\left(\frac{\partial}{\partial\hat{t}}L\left(\frac{\hat{t}-\alpha t}{1-\alpha};d\right)\right)^{-1}\Bigg|_{\hat{t}=\hat{t}(d|t)} \\
&= -\left(-\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{1-\alpha}\right)^{-1}\Bigg|_{\hat{t}=\hat{t}(d|t)} \\
&= \alpha. \qquad \text{(C.106)}
\end{aligned}
$$

Since $\hat{\omega}(d|t)=p-\hat{t}(d|t)$, $\frac{\partial}{\partial t}\hat{\omega}(d|t)=-\alpha$. Thus, we can write any uninformed type's inferred value of $\bar{\omega}(s)$ upon observing aggregate demand as

$$
\hat{\omega}(d|t)=\tilde{\omega}(d)-\alpha t, \qquad \text{(C.107)}
$$

where $\tilde{\omega}(d)$ is independent of t. While we will not explicitly solve for $\tilde{\omega}(d)$ (which will depend on F and α), we now argue that, in equilibrium, the aggregate quantity demanded by uninformed agents is equal to the aggregate quantity demanded by informed agents. To see this, we first derive the aggregate quantity demanded by uninformed agents. Since $\hat{\omega}(d|t)=\tilde{\omega}(d)-\alpha t$, an uninformed type t will demand $\tilde{\omega}(d)-p+(1-\alpha)t$ units. Thus, the truly marginal type among uninformed agents is $\hat{t} = (p - \tilde{\omega}(d))/(1-\alpha)$, and the aggregate demand among uninformed types is

$$\begin{aligned} D^U(p; \tilde{\omega}(d)) &= \int_{\hat{t}=\frac{p-\tilde{\omega}(d)}{1-\alpha}}^{\bar{t}} [\tilde{\omega}(d) - p + (1-\alpha)t] dF(t) \\ &= (1-\alpha) \int_{\hat{t}=\frac{p-\tilde{\omega}(d)}{1-\alpha}}^{\bar{t}} \left[-\frac{p-\tilde{\omega}(d)}{1-\alpha} + t \right] dF(t) \\ &= (1-\alpha) H \left(\frac{p - \tilde{\omega}(d)}{1-\alpha} \right). \end{aligned} \tag{C.108}$$

Note that $\frac{\partial}{\partial d} D^U(p; \tilde{\omega}(d)) = -H\left(\frac{p-\tilde{\omega}(d)}{1-\alpha}\right) \frac{\partial \tilde{\omega}(d)}{\partial d}$, and $\frac{\partial \tilde{\omega}(d)}{\partial d} = \frac{\partial \hat{\omega}(d|t)}{\partial d}$; hence, $\hat{t}(d|t) = p - \hat{\omega}(d|t)$ implies $\frac{\partial \tilde{\omega}(d)}{\partial d} = -\frac{\partial \hat{t}(d|t)}{\partial d}$. Since $\hat{t}(d|t)$ solves $L\left(\frac{\hat{t}-\alpha t}{1-\alpha}; d\right) = 0$, we have

$$\begin{aligned} \frac{\partial \hat{t}(d|t)}{\partial d} &= -\left(\frac{\partial}{\partial d} L\left(\frac{\hat{t} - \alpha t}{1-\alpha}; d \right) \right) \left(\frac{\partial}{\partial \hat{t}} L\left(\frac{\hat{t}-\alpha t}{1-\alpha}; d \right) \right)^{-1} \Bigg|_{\hat{t}=\hat{t}(d|t)} \\ &= -(-1) \left((1-\alpha) H\left(\frac{\hat{t}-\alpha t}{1-\alpha} \right) \frac{1}{1-\alpha} \right)^{-1} \Bigg|_{\hat{t}=\hat{t}(d|t)} \\ &= \left(H\left(\frac{\hat{t}-\alpha t}{1-\alpha} \right) \right)^{-1} \Bigg|_{\hat{t}=\hat{t}(d|t)} = \left(H\left(\frac{\hat{t}(d|t) - \alpha t}{1-\alpha} \right) \right)^{-1} \\ &= \left(H\left(\frac{p - \hat{\omega}(d|t) - \alpha t}{1-\alpha} \right) \right)^{-1} = \left(H\left(\frac{p - \tilde{\omega}(d)}{1-\alpha} \right) \right)^{-1}, \end{aligned} \tag{C.109}$$

and therefore

$$\frac{\partial \tilde{\omega}(d)}{\partial d} = -\frac{\partial \hat{t}(d|t)}{\partial d} \quad \Rightarrow \quad \frac{\partial \tilde{\omega}(d)}{\partial d} = -\left(H\left(\frac{p-\tilde{\omega}(d)}{1-\alpha} \right) \right)^{-1}, \tag{C.110}$$

which implies

$$\begin{aligned} &\frac{\partial}{\partial d} D^U(p; \tilde{\omega}(d)) = -H\left(\frac{p-\tilde{\omega}(d)}{1-\alpha} \right) \frac{\partial \tilde{\omega}(d)}{\partial d} \\ &\qquad \Rightarrow \quad \frac{\partial}{\partial d} D^U(p; \tilde{\omega}(d)) = H\left(\frac{p-\tilde{\omega}(d)}{1-\alpha} \right) \left(H\left(\frac{p-\tilde{\omega}(d)}{1-\alpha} \right) \right)^{-1} = 1. \end{aligned} \tag{C.111}$$

Thus, D^U as a function of the observed equilibrium quantity must vary identically with d; that is, $D^U(p; \tilde{\omega}(d)) = d + c$ for some constant c. But the only constant generically consistent with the required equilibrium condition of $d = \lambda D^I(p; \bar{\omega}(s)) + (1-\lambda) D^U(p; \tilde{\omega}(d))$ is $c = 0$. Thus, in equilibrium, $\tilde{\omega}(d)$ must be such that $D^U(p; \tilde{\omega}(d)) = D^I(p; \bar{\omega}(s))$, so that $d = D^I(p; \bar{\omega}(s))$. For shorthand, let $\hat{\omega}(t)$ denote $\hat{\omega}(d|t)$ evaluated at $d = D^I(p; \bar{\omega}(s))$.

Part 1. As established in (C.107), an uninformed agent with taste t forms an estimate of ω equal to $\hat{\omega}(t) = \tilde{\omega}(d) - \alpha t$, where $\tilde{\omega}(d)$ is independent of t. Thus, $\hat{\omega}(t)$ is decreasing in t whenever $\alpha > 0$.

Part 3. We prove Part 3 before Part 2. As argued above, in equilibrium $D^U(p; \tilde{\omega}(d)) = D^I(p; \bar{\omega}(s))$ must hold. Recall that $t^* = p - \bar{\omega}(s)$ and $\hat{t} = (p - \tilde{\omega}(d))/(1-\alpha)$ are the marginal informed and uninformed types, respectively. From (C.103) and (C.108), we have $D^I(p; \bar{\omega}(s)) =$

$H(t^*)$ and $D^U(p;\tilde{\omega}(d)) = (1-\alpha)H(\hat{t})$. Hence, in equilibrium, we must have $H(t^*) = (1-\alpha)H(\hat{t})$. Since H is strictly decreasing, $\hat{t} < t^*$ whenever $\alpha > 0$.

Part 2. Next, we argue that the uninformed marginal type overestimates ω; that is $\hat{t} < t^*$ if and only if $(p - \tilde{\omega}(d))/(1-\alpha) < p - \bar{\omega}(s)$ which is equivalent to

$$\tilde{\omega}(d) > (1-\alpha)\bar{\omega}(s) + \alpha p. \tag{C.112}$$

Notice that $\hat{\omega}(\hat{t}) = \tilde{\omega}(d) - \alpha\hat{t} = \tilde{\omega}(d) - \alpha(p - \tilde{\omega}(d))/(1-\alpha)$ and thus $\hat{\omega}(\hat{t}) > \bar{\omega}(s) \Leftrightarrow \tilde{\omega}(d) - \alpha p > (1-\alpha)\bar{\omega}(s)$, which holds given (C.112). Thus, $\hat{\omega}(\hat{t}) > \bar{\omega}(s)$. Furthermore, there must exist a $\tilde{t} \in (\hat{t}, \bar{t})$ such that $\hat{\omega}(\tilde{t}) = \bar{\omega}(s)$. If such a type did not exist, then the fact that $\hat{\omega}(t) = \tilde{\omega}(d) - \alpha t$ would imply that all uninformed types who buy in equilibrium overestimate $\bar{\omega}(s)$. But this, together with the fact that $\hat{t} < t^*$, would then imply that $D^U(p;\tilde{\omega}(d)) > D^I(p;\bar{\omega}(s))$ since, relative to informed types, a wider interval of uninformed types buy and they all overestimate $\bar{\omega}(s)$. Yet this contradicts the requirement that $D^U(p;\tilde{\omega}(d)) = D^I(p;\bar{\omega}(s))$, and hence there exists a $\tilde{t} \in (\hat{t}, \bar{t})$ such that $\hat{\omega}(\tilde{t}) = \bar{\omega}(s)$; moreover, $\hat{\omega}(t) = \tilde{\omega}(d) - \alpha t$ implies that $\hat{\omega}(t) > \bar{\omega}(s)$ for $t < \tilde{t}$ and $\hat{\omega}(t) < \bar{\omega}(s)$ for $t > \tilde{t}$. Since an uninformed type demands $x^*(p;\hat{\omega}(t), t) = \hat{\omega}(t) + t - p$, we additionally have $x^*(p;\hat{\omega}(t), t) > x^*(p;\bar{\omega}(s), t)$ for $t < \tilde{t}$ and $x^*(p;\hat{\omega}(t), t) < x^*(p;\bar{\omega}(s), t)$ for $t > \tilde{t}$.

Part 4. Note that $\left|x^*(p;\hat{\omega}(t), t) - x^*(p;\bar{\omega}(s), t)\right| = \left|\hat{\omega}(t) - \bar{\omega}(s)\right| = \left|\tilde{\omega}(d) - \bar{\omega}(s) - \alpha t\right|$. By definition of $\tilde{t}$, $\hat{\omega}(\tilde{t}) = \tilde{\omega}(d) - \alpha\tilde{t} = \bar{\omega}(s)$. Thus, $\left|\tilde{\omega}(d) - \bar{\omega}(s) - \alpha t\right| = \left|\tilde{\omega}(d) - [\tilde{\omega}(d) - \alpha\tilde{t}] - \alpha t\right| = \left|\alpha\tilde{t} - \alpha t\right|$, and hence $\left|x^*(p;\hat{\omega}(t), t) - x^*(p;\bar{\omega}(s), t)\right| = \alpha|t - \tilde{t}|$. ■